Lecture 5
Karnaugh Maps

- Algebraic procedures:
    - Difficult to apply in a systematic way.
    - Difficult to tell when you have arrived at a minimum solution.
- Karnaugh map (K-map) can be used to minimize functions of up to 6 variables.
  – K-map is directly applied to two-level networks composed of AND and OR gates.
    - Sum-of-products, (SOP)
    - Product-of-sum, (POS).

Minimum SOP

- It has a minimum no. of terms.
  - That is, it has a minimum number of gates.
- It has a minimum no. of gate inputs.
  - That is, minimum no. of literals.
  - Each term in the minimum SOP is a prime implicant, i.e., it cannot be combined with others.
- It may not be unique.
  - Depend on the order in which terms are combined or eliminated.

Minimum SOP

- Example: vertical input scheme

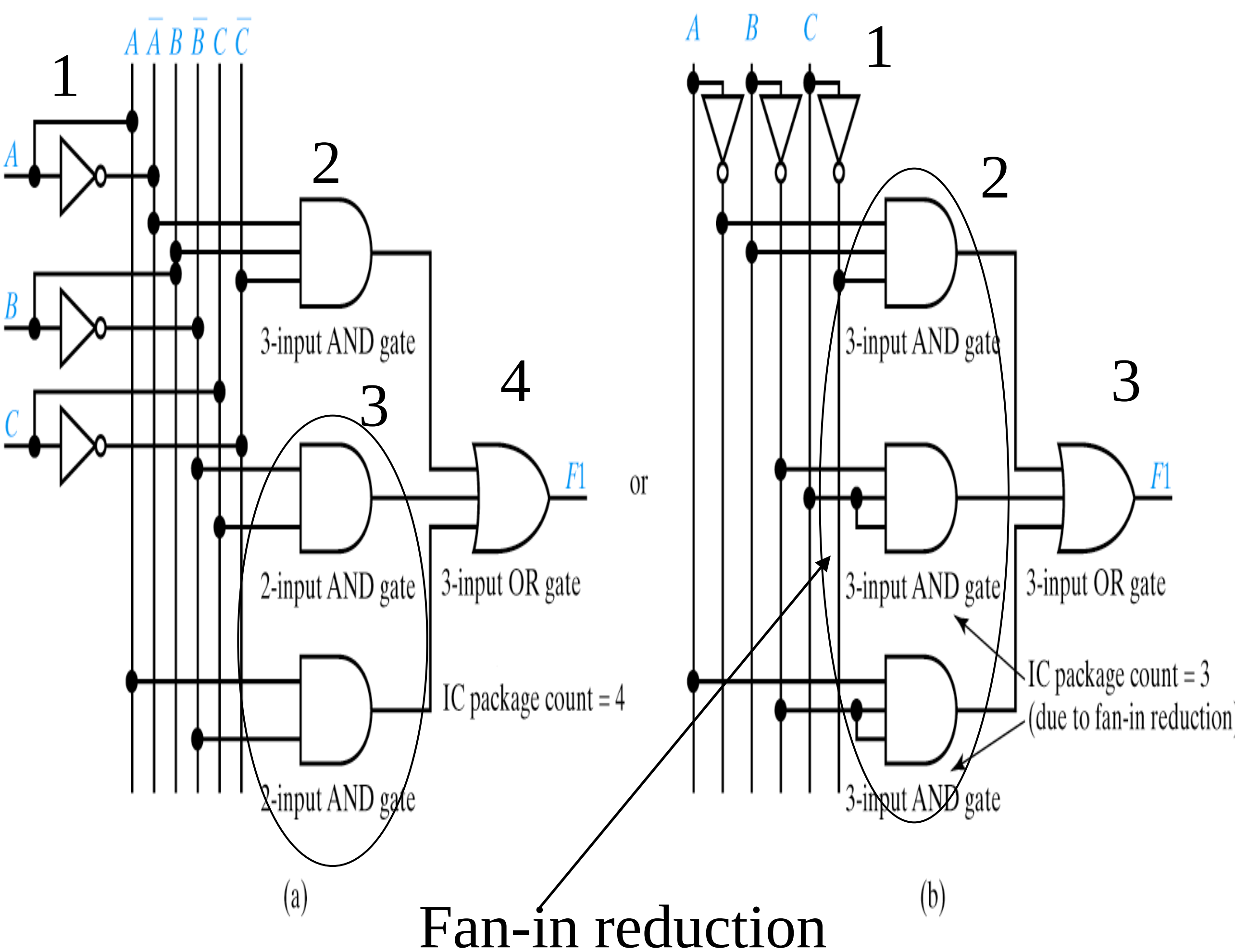


Figure 3.2.5 IC logic circuit designs for a minimum SOP form of a function using a vertical input scheme: (a) without fan-in reduction, (b) with fan-in reduction.

Minimum POS

- It has a minimum no. factors.
- It has a minimum no. of literals.
- It may not be unique.
  - Use (X+Y) (X+Y’) = X
  - Use (X +C) (X’ + D)(C+D) = (X+C)(X’+D) to eliminate term.

Minimum POS

- Example: Vertical input scheme

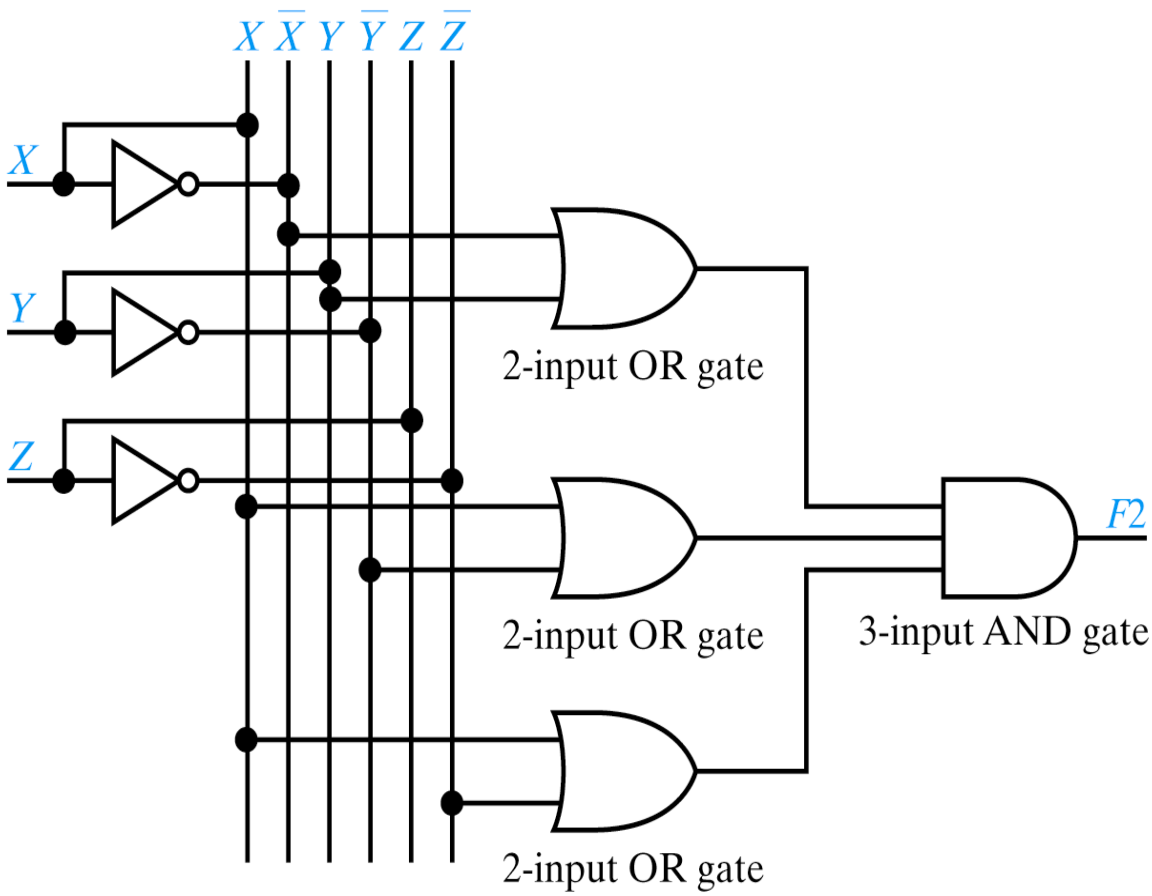
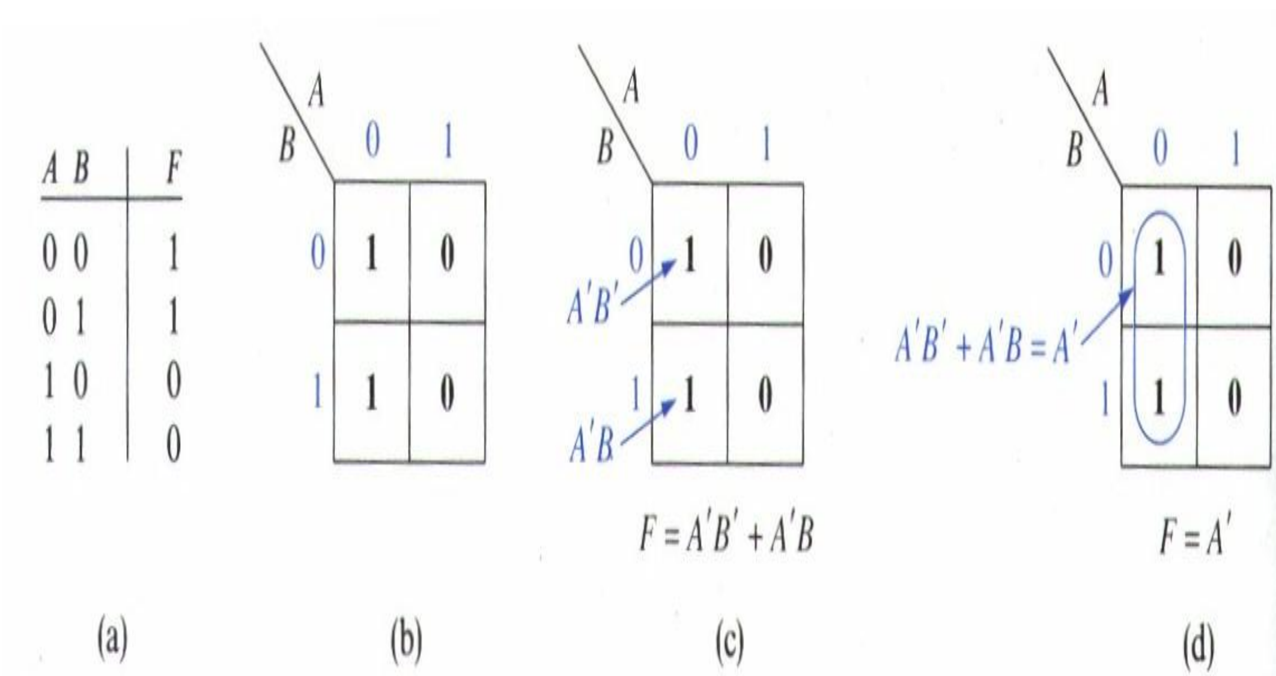


Figure 3.2.7 IC logic circuit design for a minimum POS form of a function using a vertical input scheme.

2-Variable K-map

– Place 1s and 0s from the truth table in the K-map.
– Each square of 1s = minterms.
– Minterms in adjacent squares can be combined since they differ in only one variable. Use XY’ + XY = X.

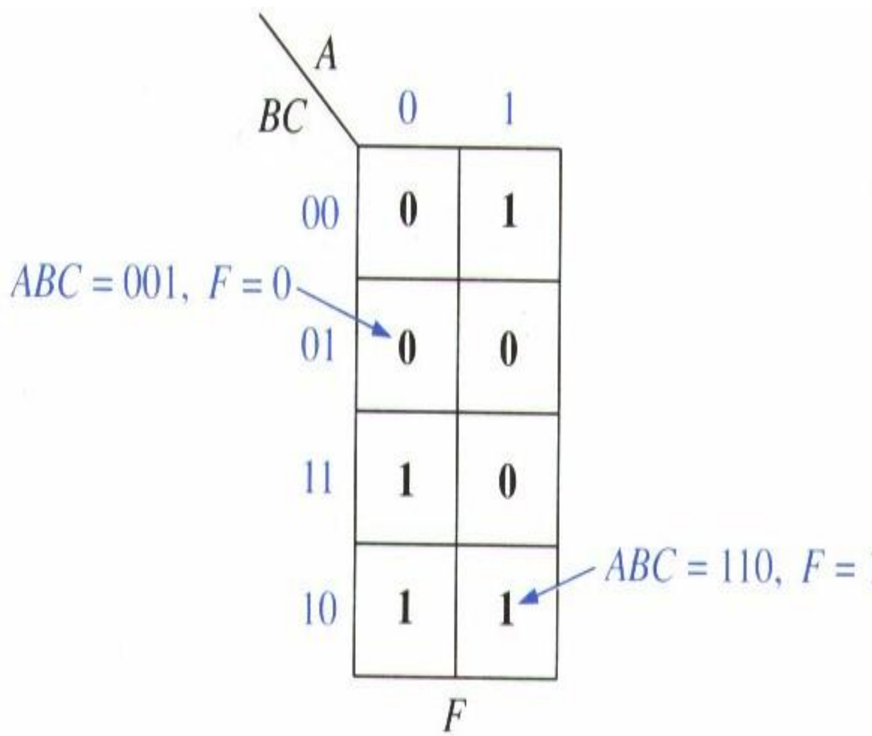


3-Variable K-map

- Note BC is listed in the order of 00, 01, 11, 10. (Gray code)
- Minterms in adjacent squares that differ in only one variable can be combined using XY' + XY = X.

A B C	F
0 0 0	0
0 0 1	0
0 1 0	1
0 1 1	1
1 0 0	1
1 0 1	0
1 1 0	1
1 1 1	0

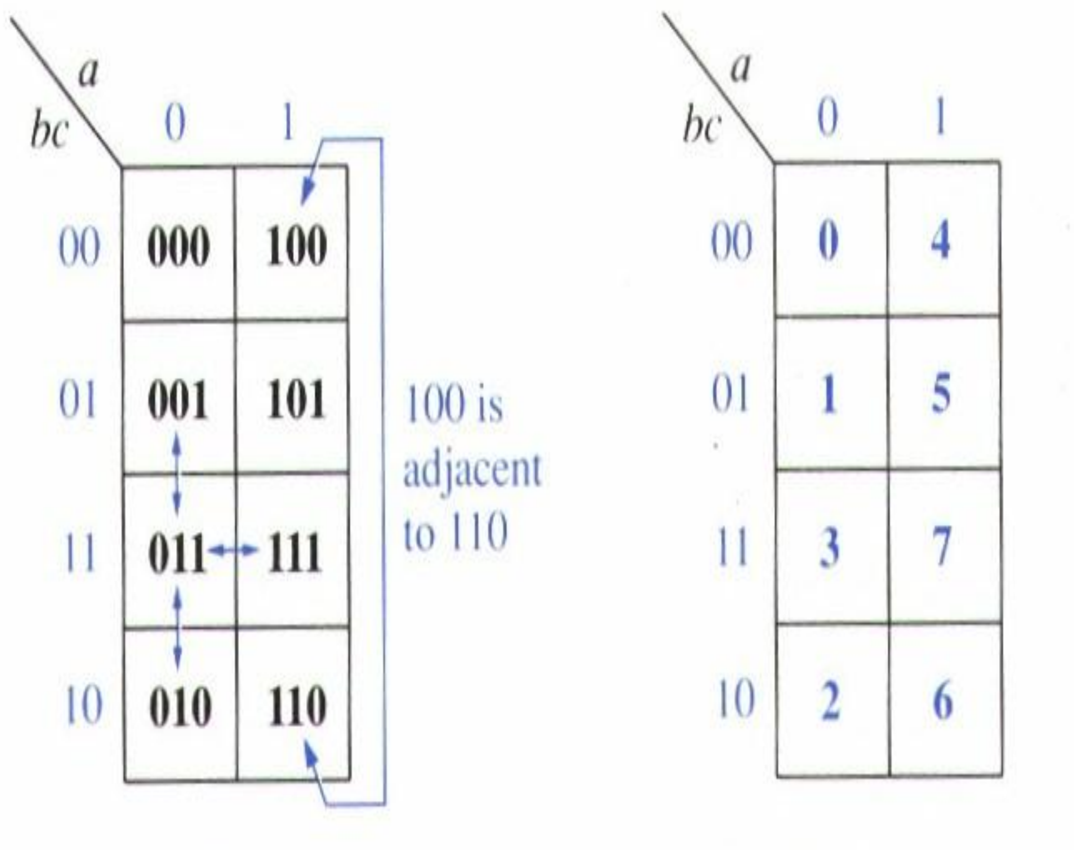
(a)



(b)

Location of Minterms

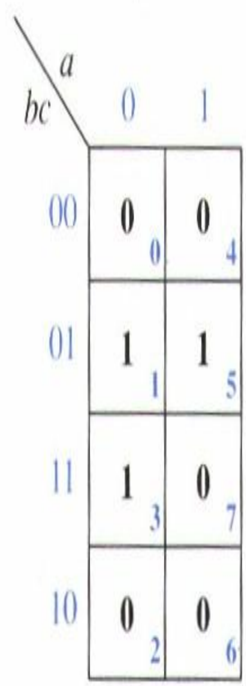
– Adjacent terms in 3-variable K map.



(a) Binary notation (b) Decimal notation

K Map Example

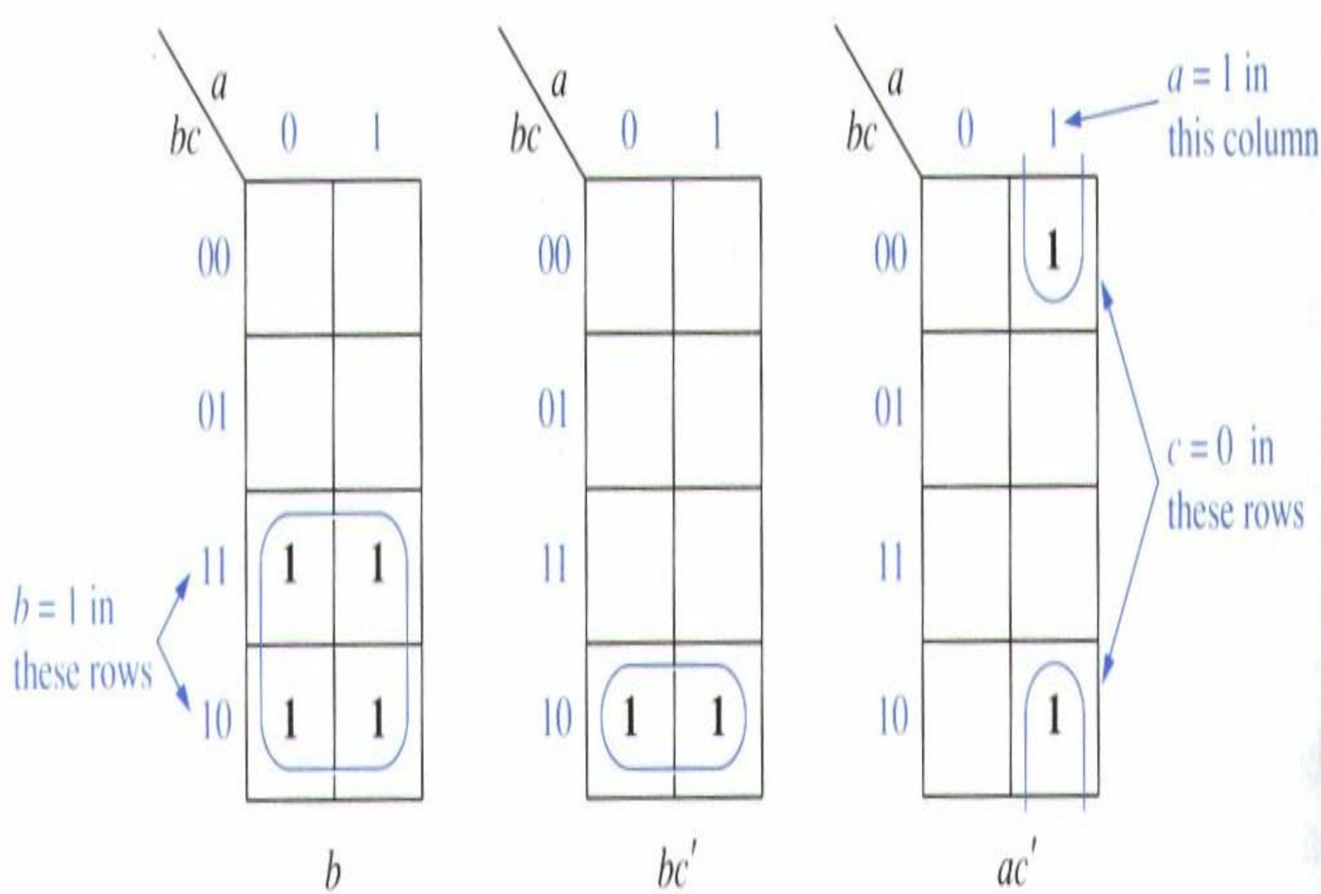
– K-map of $F(a,b,c) = \sum m(1,3,5)$ $= \prod M(0,2,4,6,7)$



Karnaugh Map of
$F(a, b, c) = \sum m(1, 3, 5) = \prod M(0, 2, 4, 6, 7)$

Place Product Terms on K Map

- Example
  – Place b, bc’ and ac’ in the 3-variable K map.



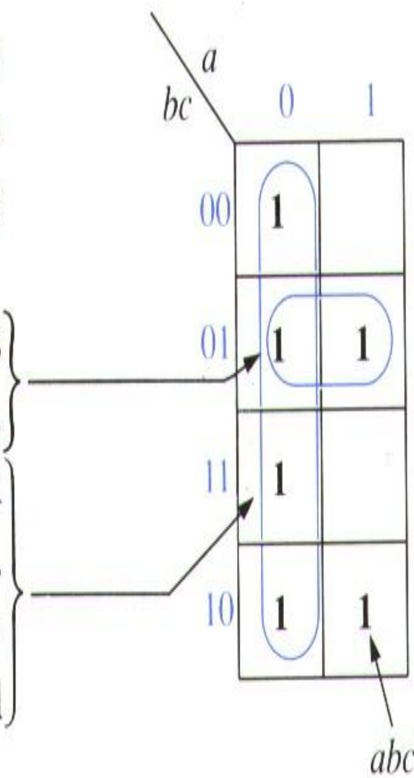
More Example

– Exercise. Plot f(a, b, c) = abc’ + b’c + a’ into the K-map.

$$f(a, b, c) = abc' + b'c + a'$$

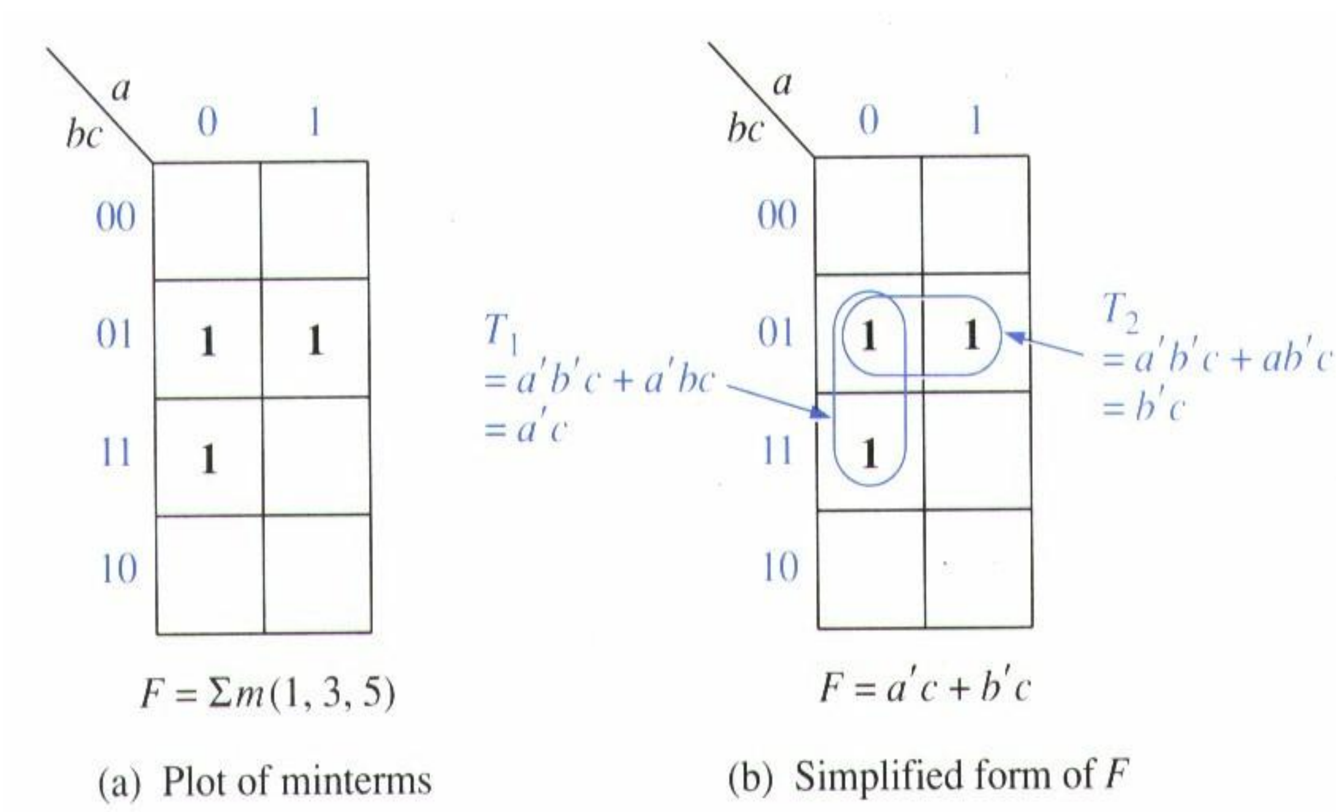
we would plot the map as follows:

1. The term abc' is 1 when $a = 1$ and $bc = 10$, so we place a 1 in the square which corresponds to the $a = 1$ column and the $bc = 10$ row of the map.
2. The term $b'c$ is 1 when $bc = 01$, so we place 1’s in both squares of the $bc = 01$ row of the map.
3. The term a' is 1 when $a = 0$, so we place 1’s in all the squares of the $a = 0$ column of the map. (*Note:* since there already is a 1 in the abc = 001 square, we do not have to place a second 1 there because $x + x = x$.)



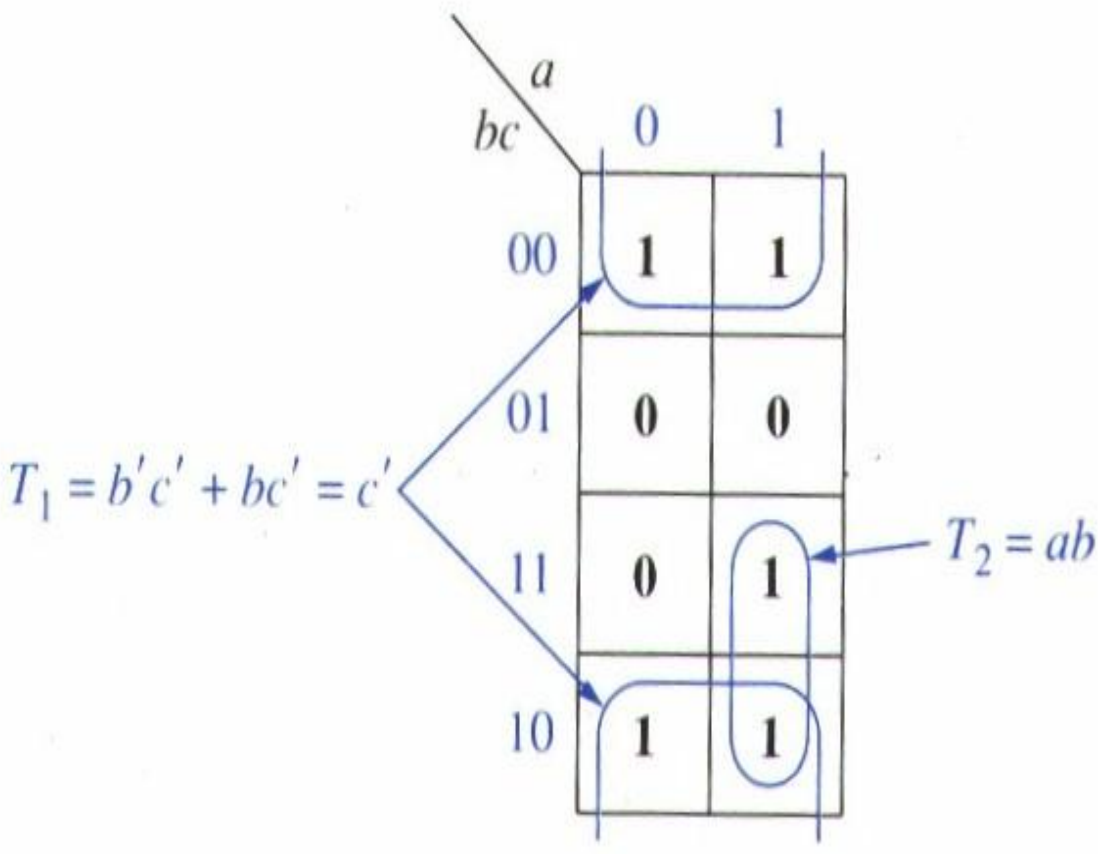
Simplication Example

- Exercise. Simplify: F(a,b,c) = ∑m(1,3,5)
  - Procedure: place minterms into map.
  - Select adjacent 1's in group of two 1's or four 1's etc.
  - Kick off x and x'.



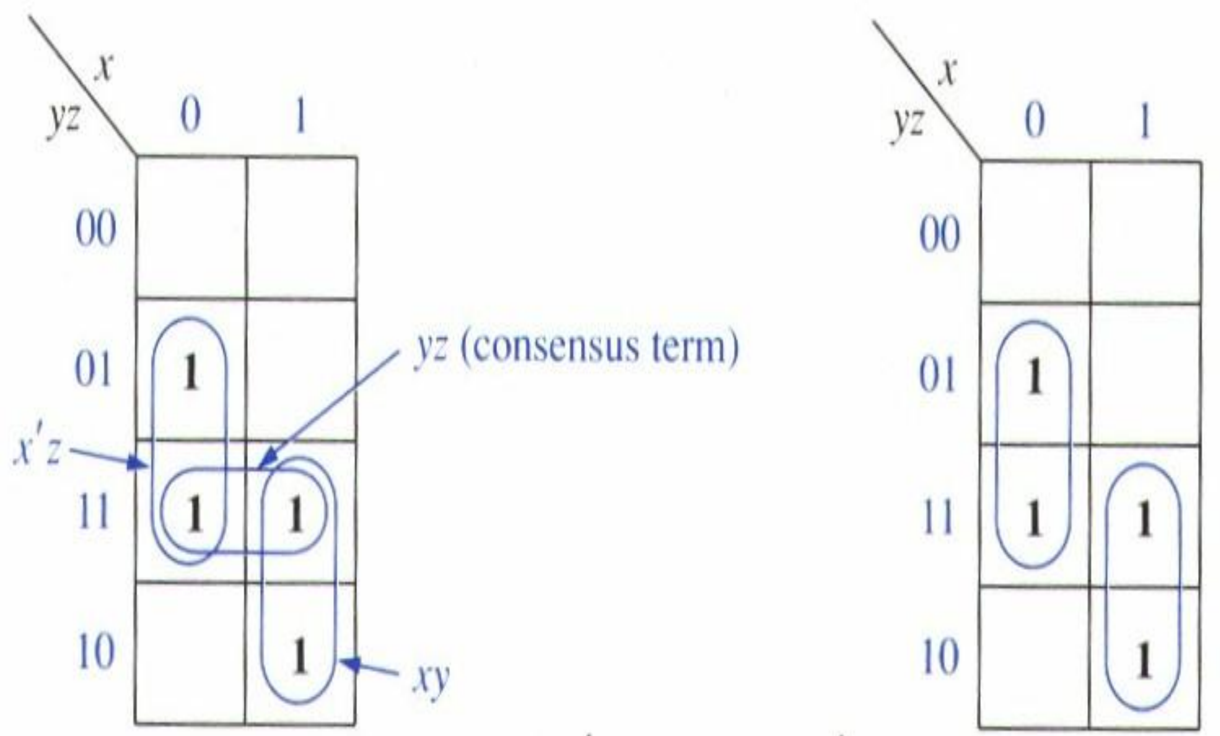
More Example

– The complement of F
  - Using four 1's to eliminate two variables.



Redundant Terms

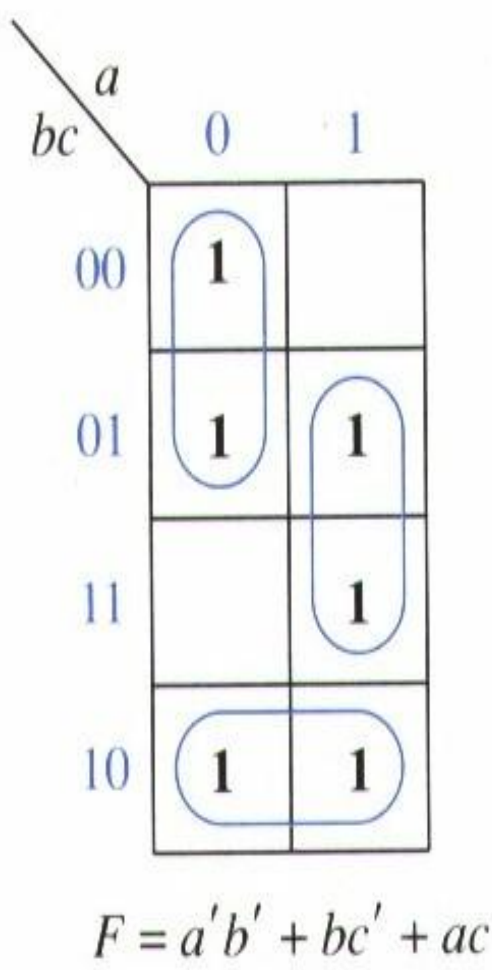
- If a term is covered by two other terms, that term is redundant. That is, it is a consensus term.
- Example: yz is the redundant.

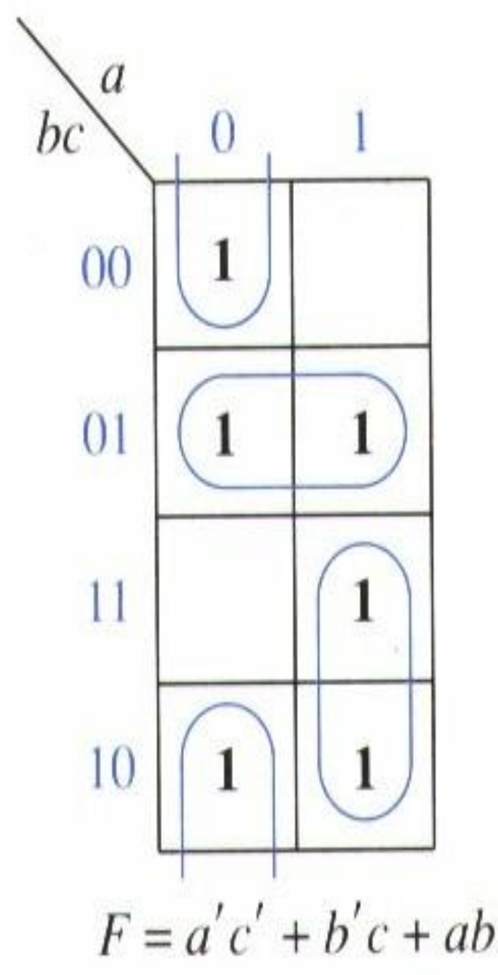


$$xy + x'z + yz = xy + x'z$$

More Than Two Minimum Solutions

- $F = \sum m(0,1,2,5,6,7)$





4-Variable K Map

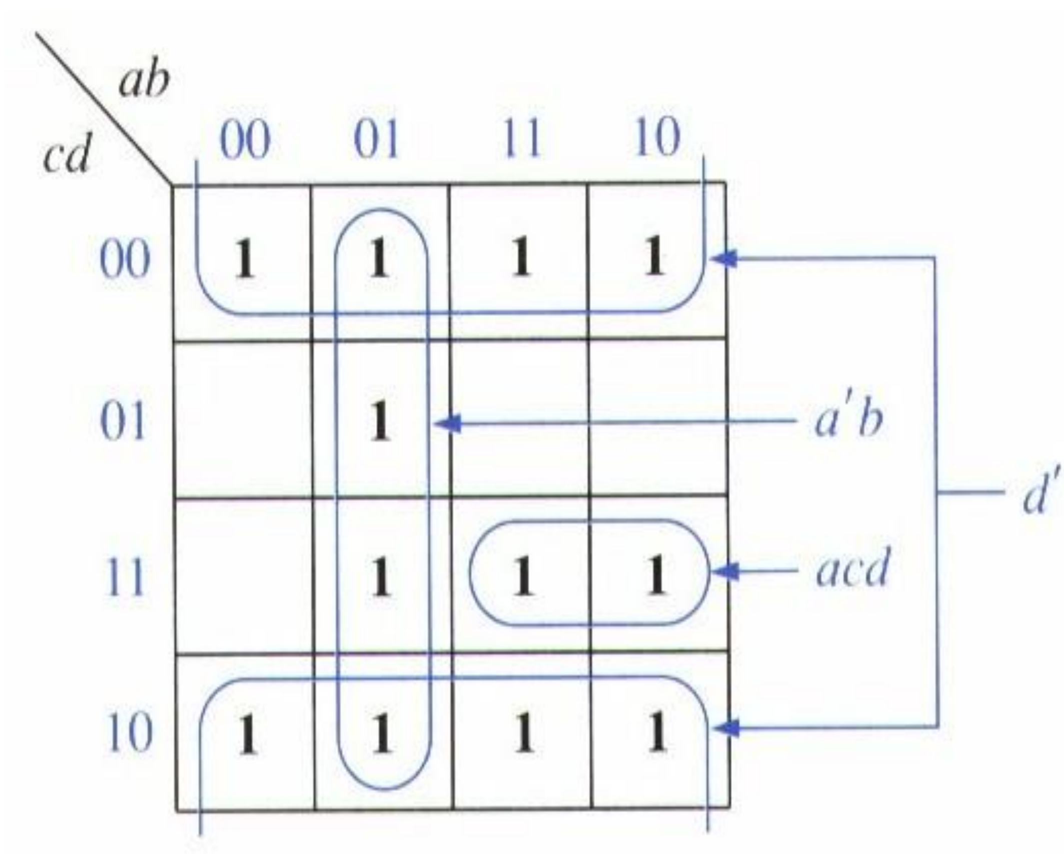
– Each minterm is adjacent to 4 terms with which it can combine.
  - 0, 8 are adjacent squares
  - 0, 2 are adjacent squares, etc.
  - 1, 4, 13, 7 are adjacent to 5.

CD \ AB	00	01	11	10
00	0	4	12	8
01	1	5	13	9
11	3	7	15	11
10	2	6	14	10

Plot a 4-variable Expression

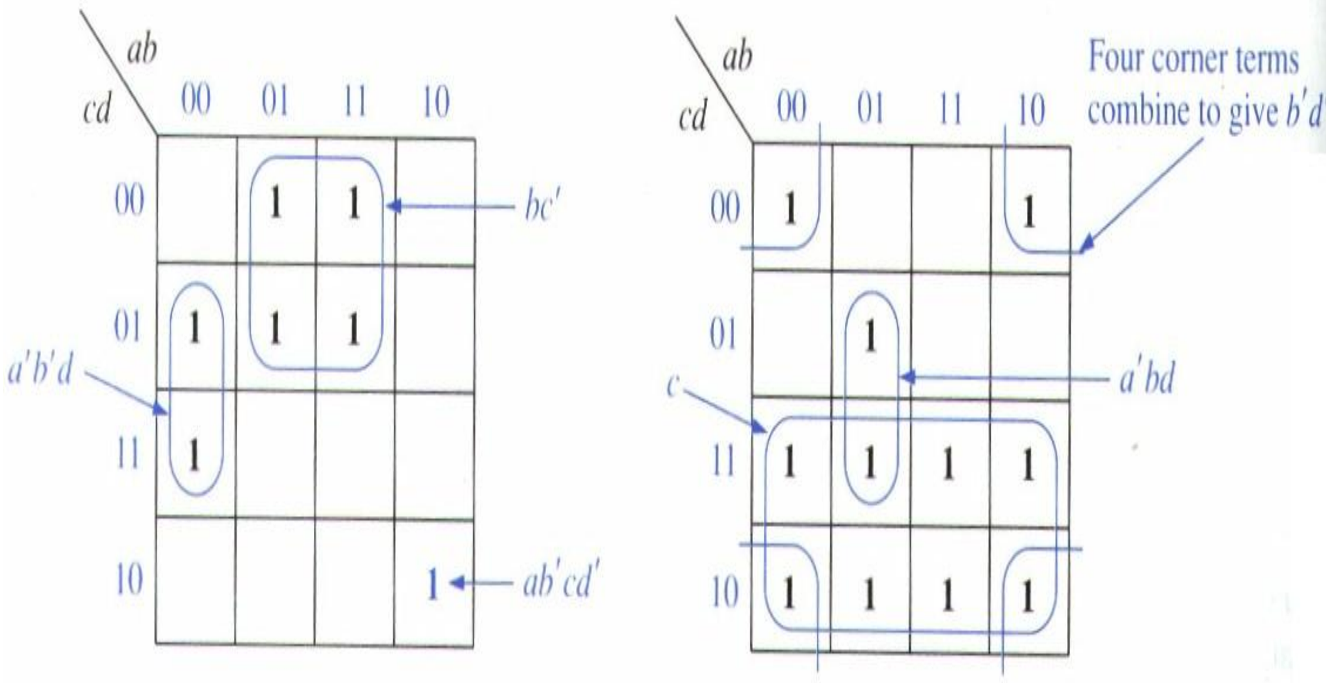
- F(a,b,c,d) = acd + a'b + d'

  acd = 1 if a=1, c=1, d=1



Simplification Example

- Minterms are combined in groups of 2, 4, or 8 to eliminate 1, 2, 3 variables.
- Corner terms.



$f_1 = \Sigma m(1, 3, 4, 5, 10, 12, 13)$
$= bc' + a'b'd + ab'cd'$

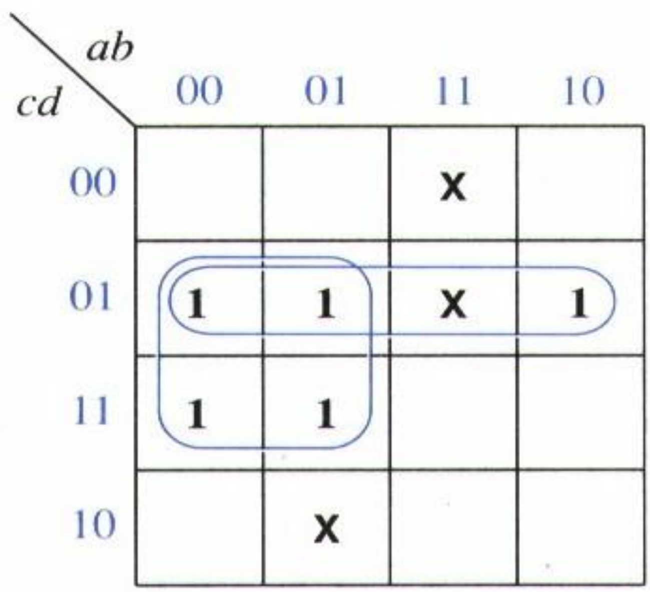
(a)

$f_2 = \Sigma m(0, 2, 3, 5, 6, 7, 8, 10, 11, 14, 15)$
$= c + b'd' + a'bd$

(b)

Simplification with Don’t Care

- Don’t care “x” is covered if it helps. Otherwise leave it along.



$$f = \Sigma m(1, 3, 5, 7, 9) + \Sigma d(6, 12, 13)$$
$$= a'd + c'd$$

Get a Minimum POS Using K Map

- Cover 0's to get simplified POS.
  - We want 0 in each term.

$$f = x'z' + wyz + w'y'z' + x'y$$

First, the 1's of f are plotted in Fig. 5–14. Then, from the 0's,

$$f' = y'z + wxz' + w'xy$$

and the minimum product of sums for f is

$$f = (y + z')(w' + x' + z)(w + x' + y')$$

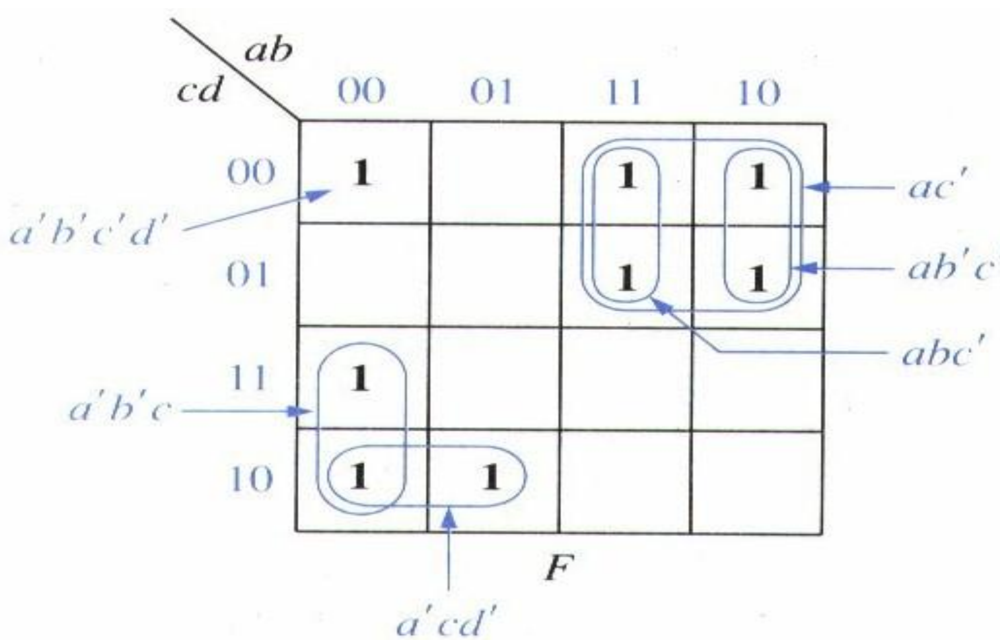
yz \ wx	00	01	11	10
00	1	1	0	1
01	0	0	0	0
11	1	0	1	1
10	1	0	0	1

Determination of Minimum Expressions Using Essential Prime Implicants

- Definitions:
  - **Implicants**: An implicant of a function F is a single element of the on set (1) or any group of elements that can be combined together in a K-map.
  - **Prime Implicants**: An implicant that cannot be combined with another to eliminate a literal.
  - **Essential Prime Implicants**: If a particular element of the on-set is covered by a single prime implicant. That implicant is called an essential prime implicant.
- All essential primes must be part of the minimized expression.

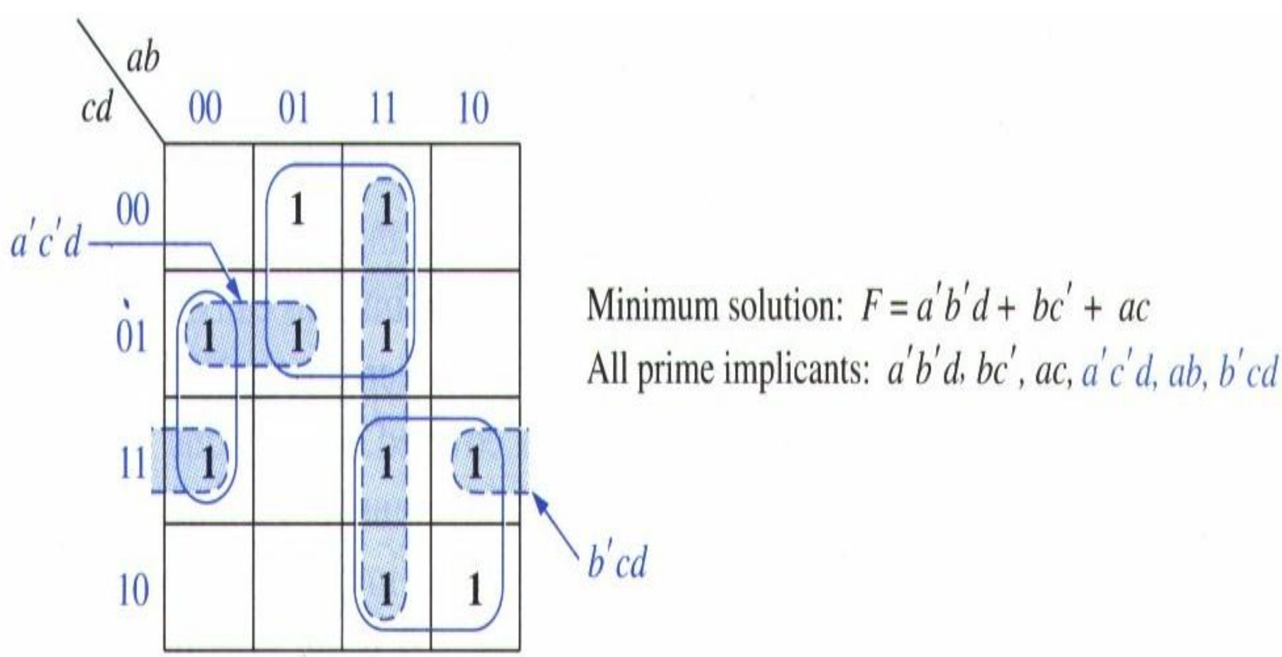
Implicant of F

- Implicant
  - Any single 1 or any group of 1's
- Prime implicant
  - An implicant that can not be combined with another term to eliminate a variable.
    - a'b'c, a'cd', ac' are prime implicant.
    - a'b'c'd' is not (combined with a'b'cd'). abc' and ab'c' are not.



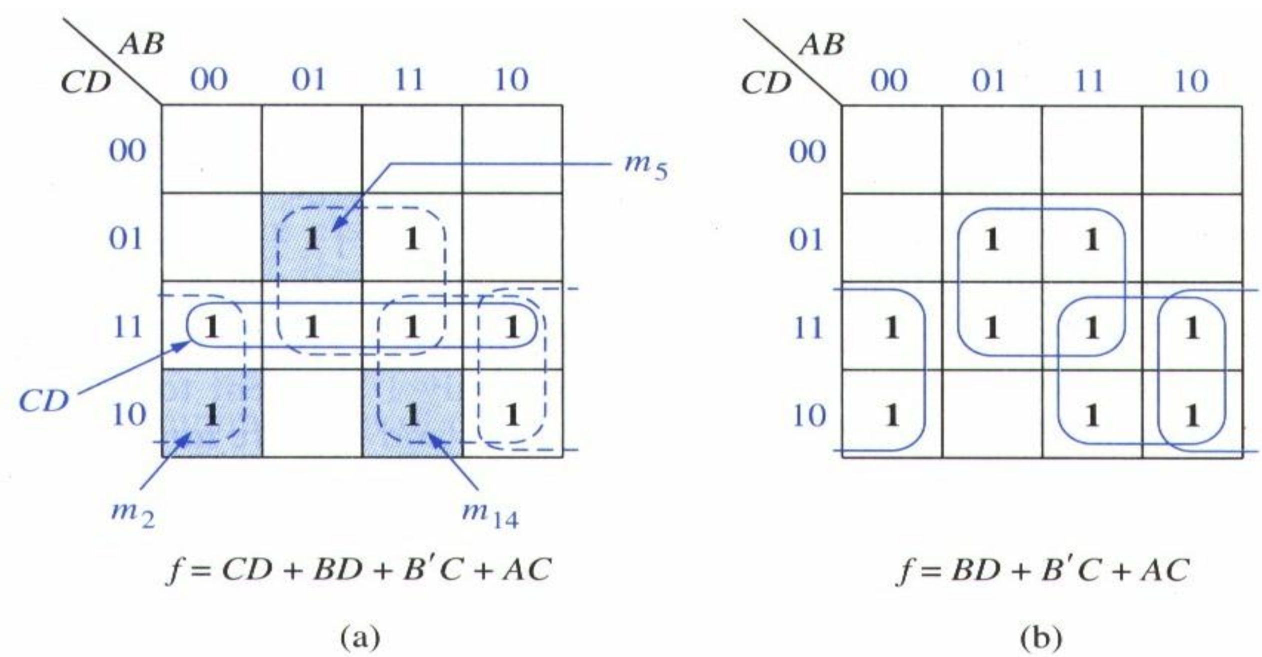
Prime Implicant

- A single 1 which is not adjacent to any other 1's.
- Two adjacent 1's which are not contained in a group of four 1's. And so on.
  - Shaded loops are also prime implicants, but not part of the minimum solution.
  - a'c'd and b'cd are already covered by other group. So we do not need them.



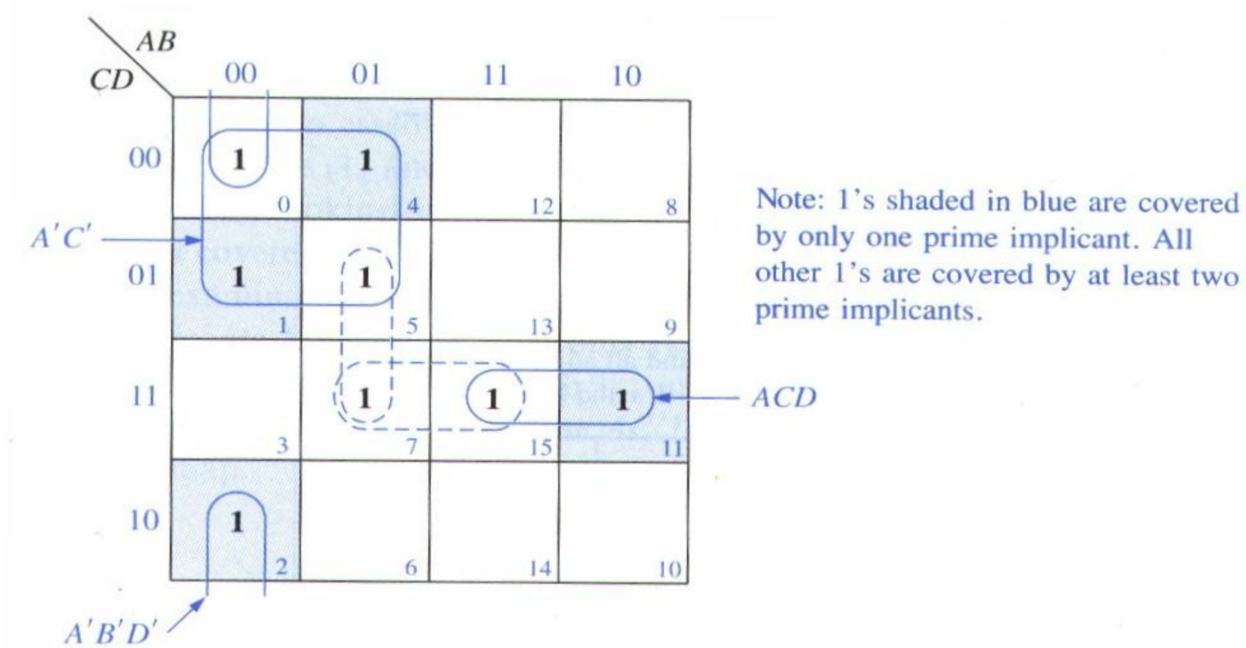
Essential Prime Implicants for Minimum SOP

- If CD is chosen first, then f has 4 terms. We don't need CD since it is covered by other group. CD is not a essential prime implicant.
- m2 is essential prime implicant since it is covered only be one prime implicant. So does m5, and m14. We need them in the answer.

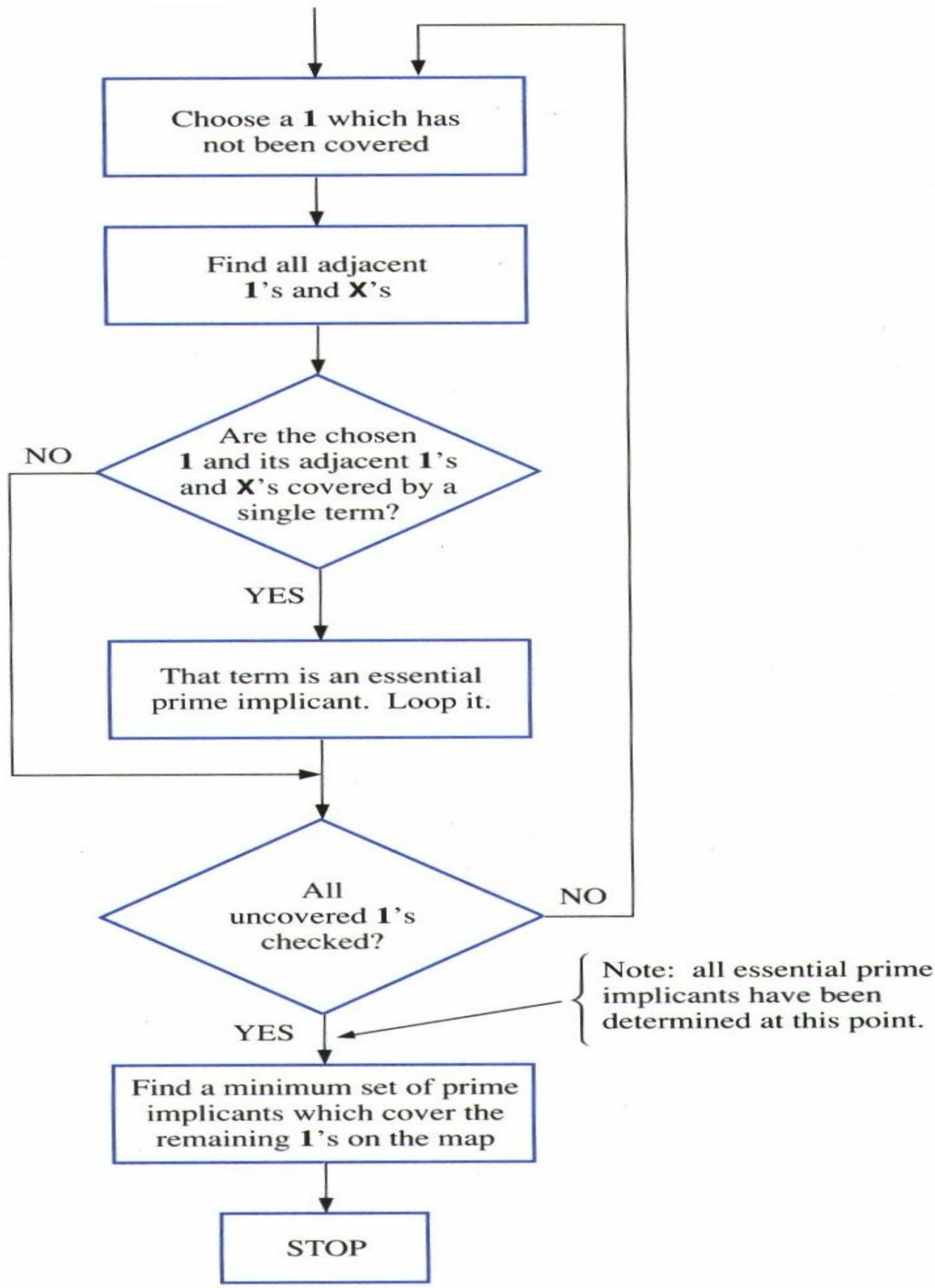


Rule of Thumb

– Look at all squares adjacent to a minterm.
  – If the given minterm and all of the 1's adjacent to it are covered by a single term, then that term is an essential prime implicant
  – But if it is covered by more than two prime implicant, we can not tell whether this term is essential or not.
  – The solution is A'C' + A'B'D' +ACD + (A'BD or BCD).



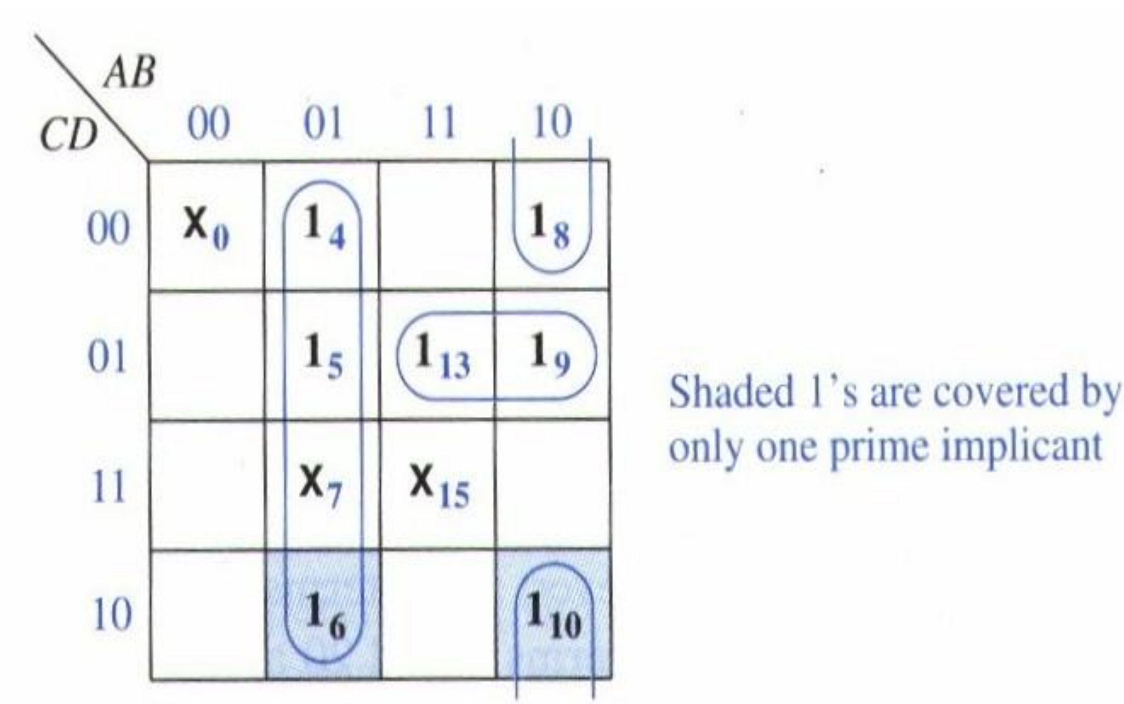
Flow Chart



Choose a 1 which has not been covered
Find all adjacent 1's and X's
Are the chosen 1 and its adjacent 1's and X's covered by a single term?
NO
YES
That term is an essential prime implicant. Loop it.
All uncovered 1's checked?
NO
YES
Note: all essential prime implicants have been determined at this point.
Find a minimum set of prime implicants which cover the remaining 1's on the map
STOP

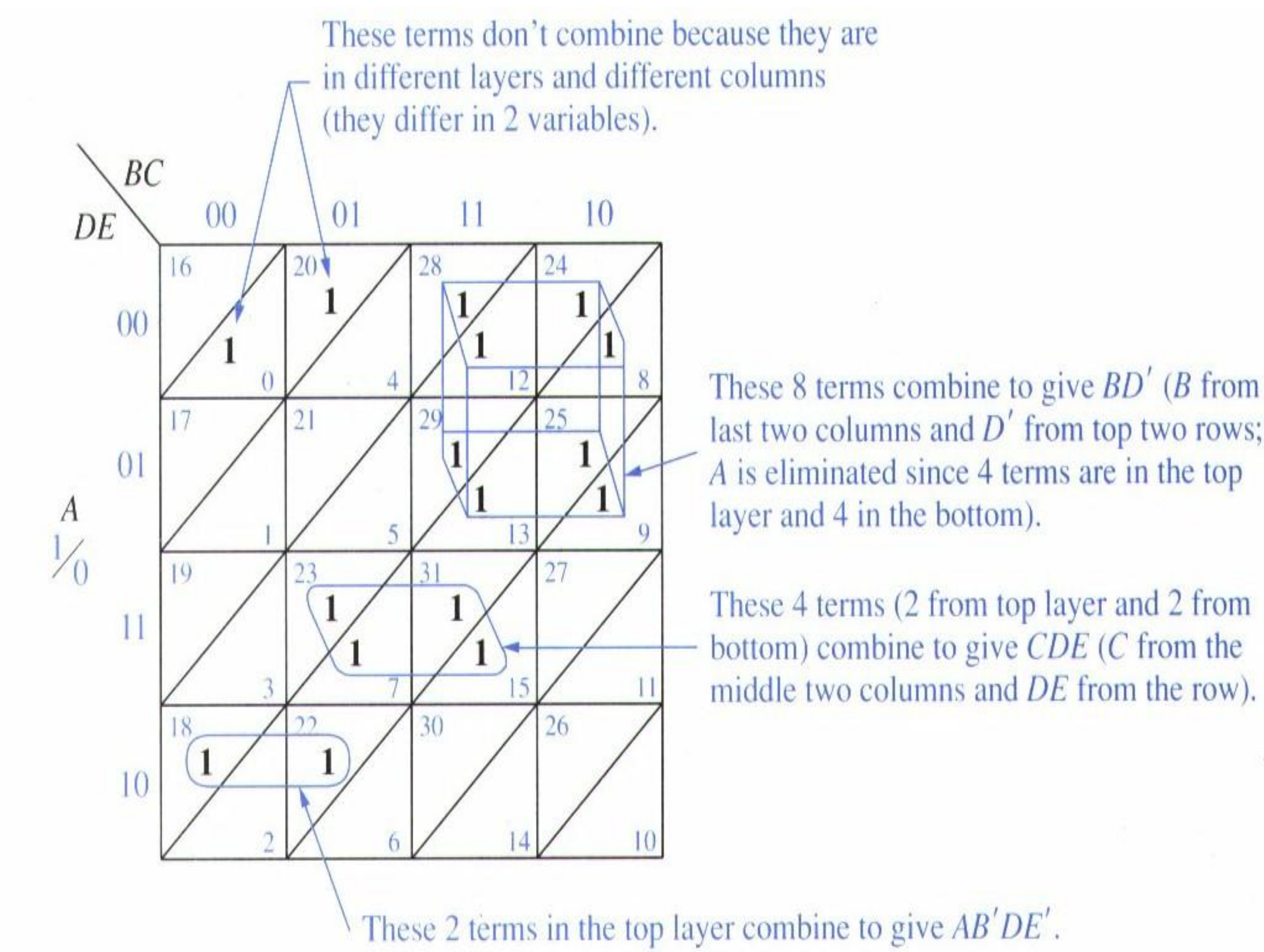
Example

- Find the 1 that is covered by only one term first (Do not share with other circle).



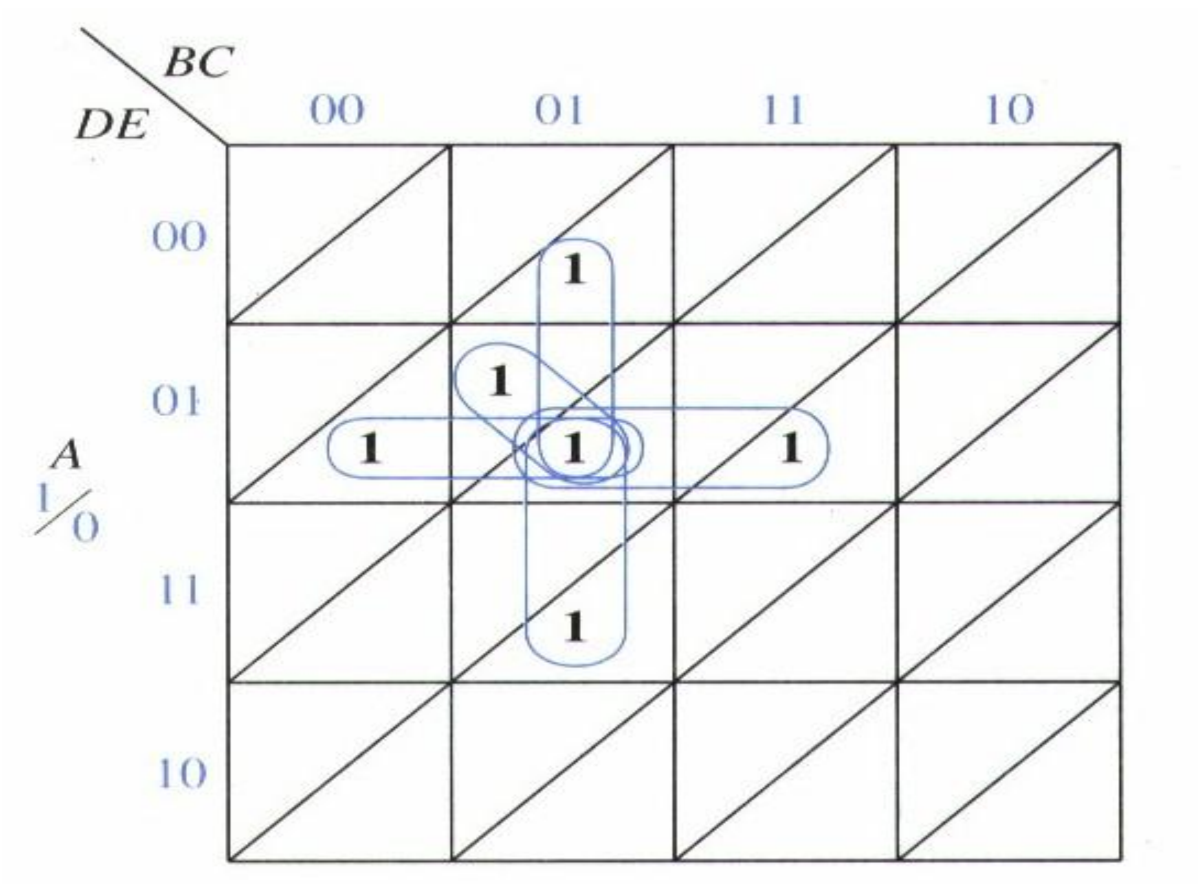
5-Variable K Map

– Use two 4-variable map to form a 5-variable K map (16 + 16 = 32) (A,B,C,D,E)
    - A’ in the bottom layer
    - A in the top layer.



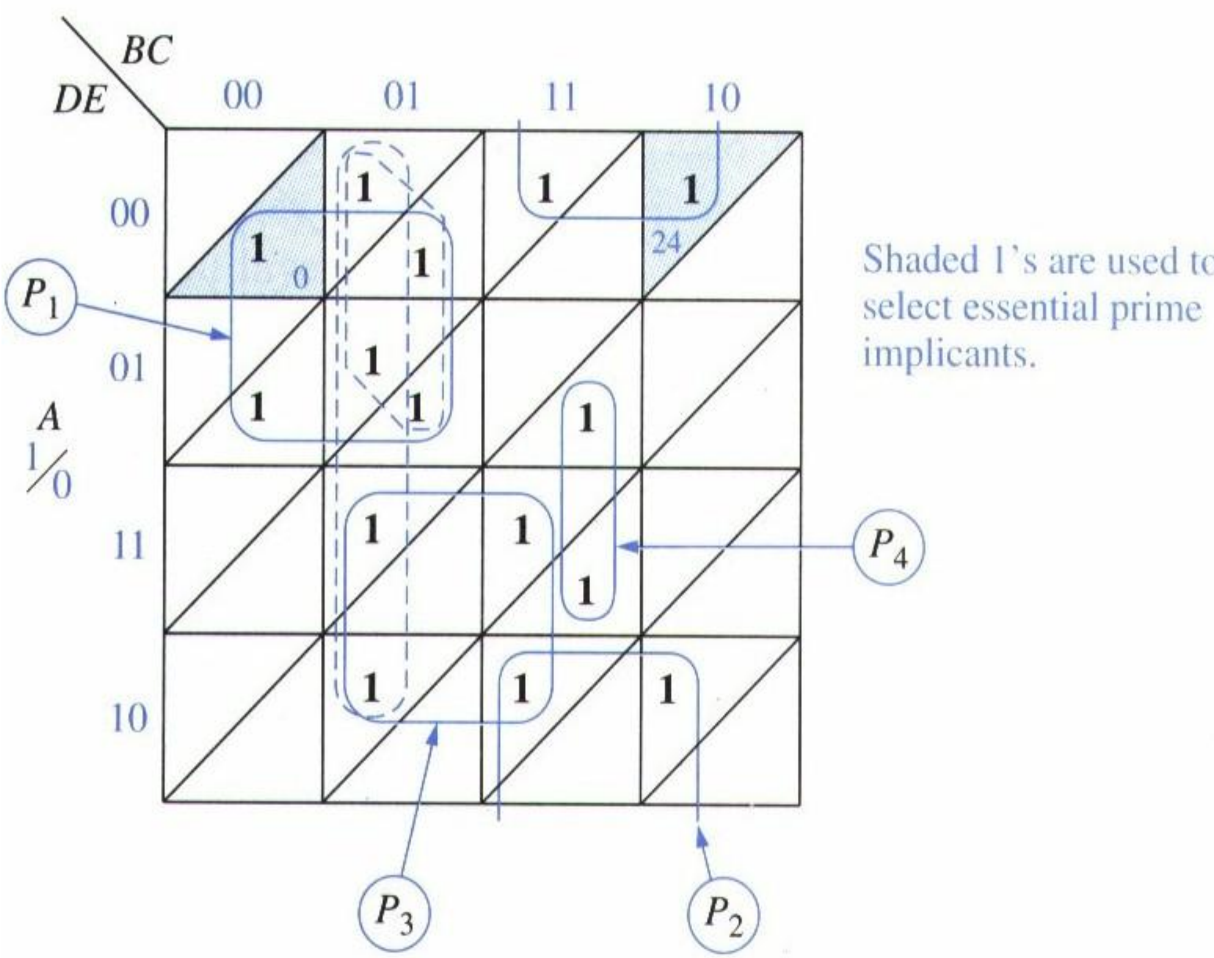
5 Neighbors

- Same plane and above or under



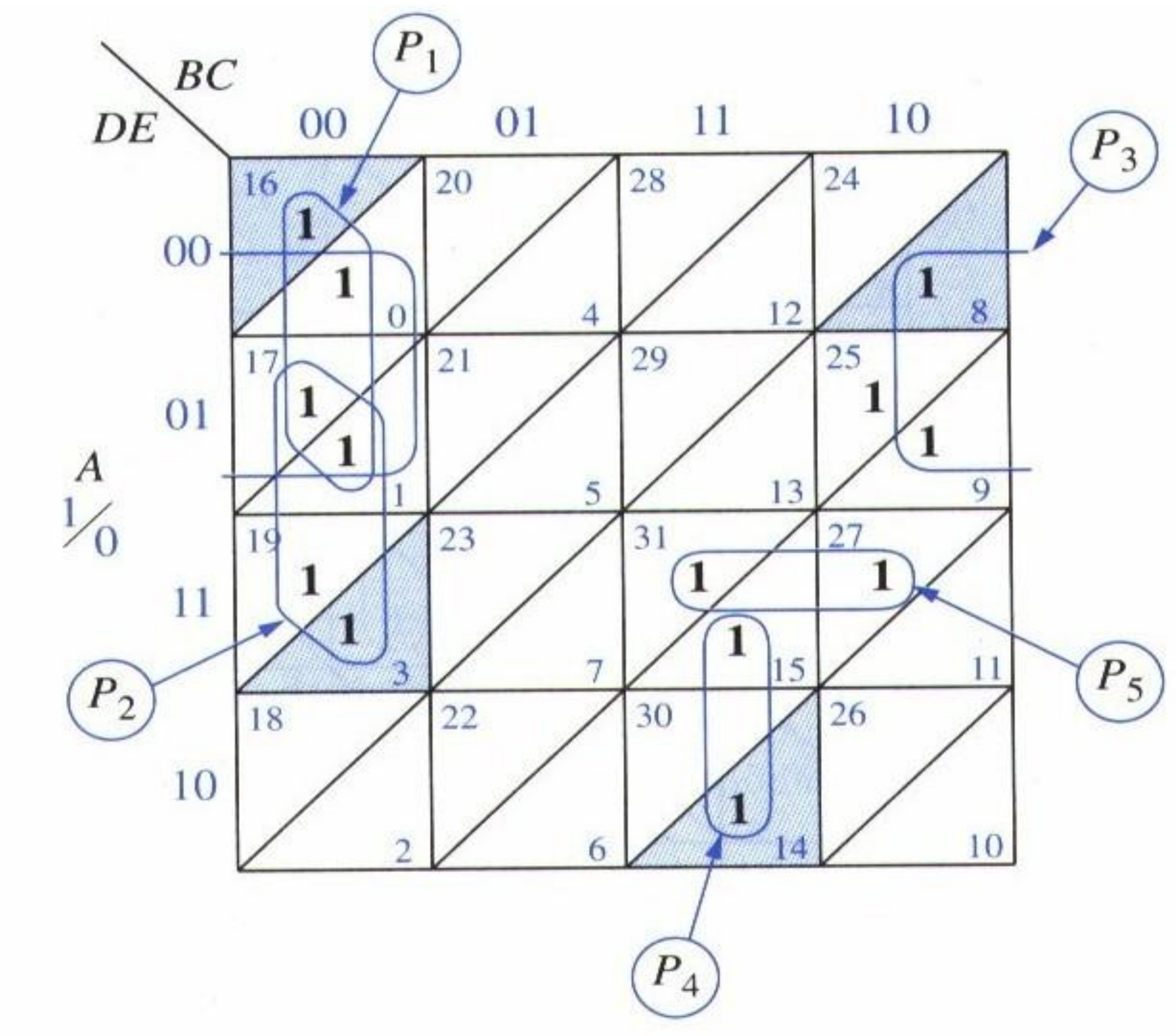
Example: 5-variables

- Ans: F = A'B'D' + ABE' + ACD + A'BCE + {AB'C or B'CD'}
  - P1 + P2 + P3 + P4 + AB'C or B'CD'



One More

- F = B’C’D’ + B’C’E + A’C’D’ + A’BCD + ABDE + {C’D’E or AC’E}
- (17,19,25,27 =AC’E), (1,9,17,25 = C’D’E)

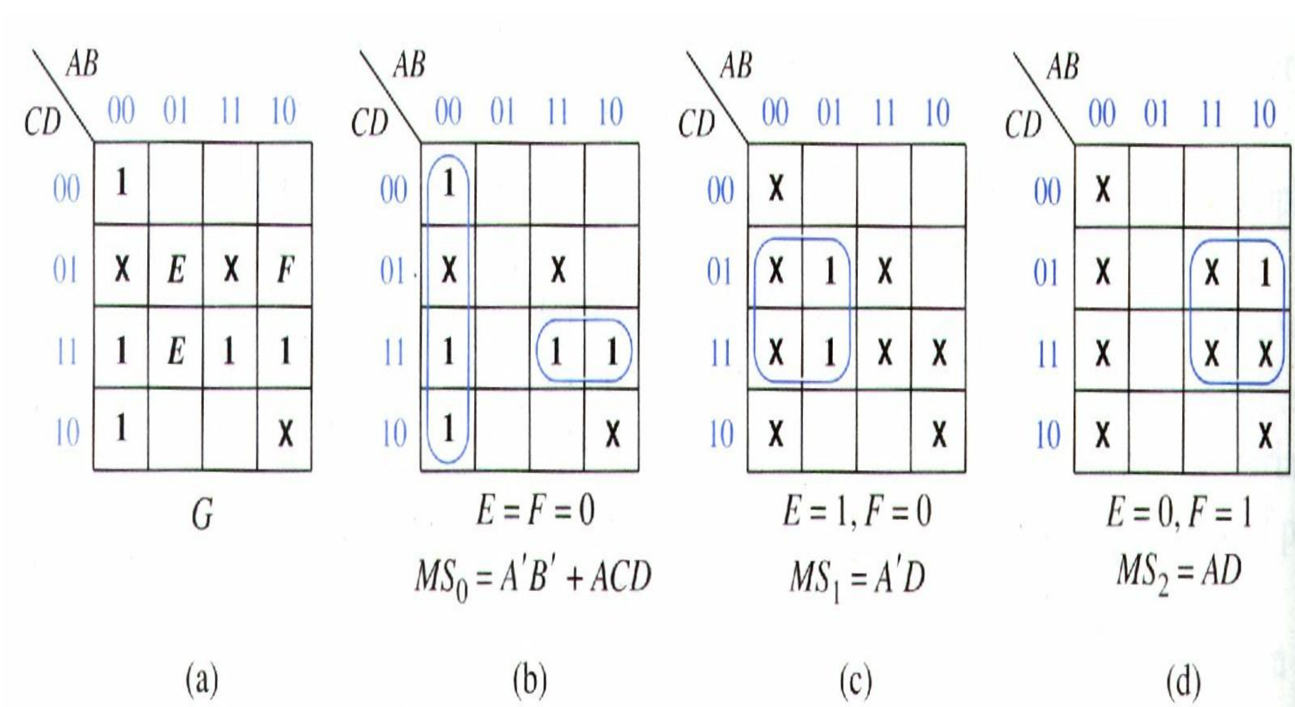


Simplification Using Map-Entered Variables

– Extend K-map for more variables.

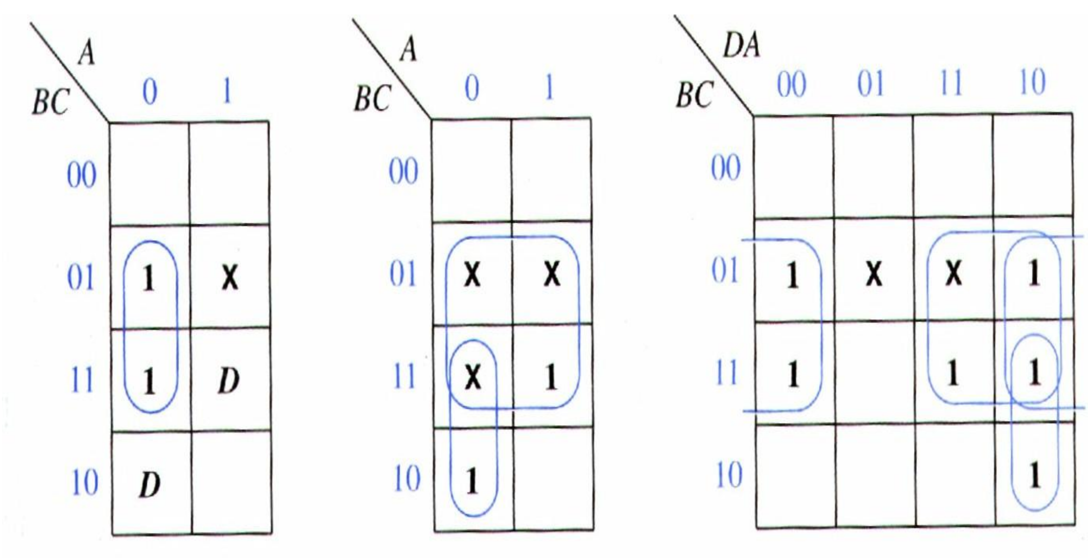
– When E appears in a square, if E = 1, then the corresponding minterm is present in the function G.

– G (A,B,C,D,E,F) = $m_0 + m_2 + m_3 + Em_5 + Em_7 + Fm_9 + m_{11} + m_{15}$ + (don't care terms)



Map-Entered Variable

- F(A,B,C,D) = A'B'C + A'BC + A'BC'D + ABCD + (AB'C), (don't care)
  - Choose D as a map-entered variable.
  - When D = 0, F = A'C (Fig. a)
  - When D = 1, F = C + A'B (Fig. b)
    - two 1's are changed to x's since they are covered in Fig. a.
- F = A'C + D(C+A'B) = A'C + CD + A'BD



(a) (b) (c)

General View for Map-Entered Variable Method

- Given a map with variables P1, P2 etc, entered into some of the squares, the minimum SOP form of F is as follows:
- $F = MS0 + P1\ MS1 + P2MS2 + \ldots$ where
  - MS0 is minimum sum obtained by setting $P1 = P2 .. = 0$
  - MS1 is minimum sum obtained by setting $P1 = 1$, $Pj = 0$ ($j \neq 1$), and replacing all 1's on the map with don't cares.

  - Previously, $G = A'B' + ACD + EA'D + FAD$.