Lecture 4
Minterm and Maxterm

- Conversion of English sentences to Boolean equations.
- Example
  - The alarm will ring iff the alarm switch is on and the door is not closed or it is after 6 PM and the window is not closed.
  - Boolean equation
    - Z = AB’ + CD’
    - If Z = 1 , the alarm will ring.
    - Draw the network. Z will drive the alarm.

Combinational Network Using a Truth Table

- Problem statement
  - ❖ Input 3 bits A, B, C = Binary number N. Output f. Output f = 1 if N >= 011 and f = 0 if N < 011.

A B C	f
000	0
001	0
010	0
011	1
100	1
101	1
110	1
111	1

Derive Algebraic Expression from Truth Table

- Using f = 1gives the SOP form.

  f = A'BC +AB'C' + AB'C + ABC' + ABC

  = A'BC + AB' +AB = A'BC + A

  = A + BC.

- Using f = 0 gives the POS form.

  Maxterms are multiplied together so that if any one of them is 0, f will be 0. See what happens if uses OR.

  f = (A+B+C)(A+B+C')(A+B'+C)

  = (A+B)(A+B'+C) = A+BC

Minterm and Maxterm

- Minterm
  - A minterm of n variables = product of n literals in which each variable appears exactly once either in T or F form, but not in both. (Also known as a standard product term)
  - Each minterm has value 1 for exactly one combination of values of variables. E.g. ABC (111) => m_7
  - A function can be written as a sum of minterms, which is referred to as a minterm expansion or a standard sum of products.

Minterm/Maxterm

- Three variables

Row No.	A B C	Minterms	Maxterms
0	0 0 0	$A'B'C' = m_0$	$A+B+C = M_0$
1	0 0 1	$A'B'C = m_1$	$A+B+C' = M_1$
2	0 1 0	$A'BC' = m_2$	$A+B'+C = M_2$
3	0 1 1	$A'BC = m_3$	$A+B'+C' = M_3$
4	1 0 0	$AB'C' = m_4$	$A'+B+C = M_4$
5	1 0 1	$AB'C = m_5$	$A'+B+C' = M_5$
6	1 1 0	$ABC' = m_6$	$A'+B'+C = M_6$
7	1 1 1	$ABC = m_7$	$A'+B'+C' = M_7$

Minterm Notation

- f = A'BC + AB'C' + AB'C + ABC' +ABC;

  The other way to represent f is:

  $f(A,B,C) = m_3 + m_4 + m_5 + m_6 + m_7$ or

  $f(A,B,C) = \sum m(3,4,5,6,7)$

  Another view,

  $f(A,B,C) = 0.m_0 + 0.m_1 + 0.m_2 + 1.m_3 + 1.m_4 + 1.m_5 + 1.m_6 + 1.m_7$

✓ Minterms present in f correspond with the 1's of f in the truth table.

Maxterm

- Maxterm
  - A maxterm of n variables = sum of n literals in which each variable appears exactly once in T or F from, but not in both.
  - Each maxterm has a value of 0 for exactly one combination of values of variables. E. g. A + B + C' (001) => M_1 (the value is 0). Therefore $M_i = m'_i$.
  - A function can be written as a product of maxterms, which is referred to as a maxterm expansion or a standard product of sums.

Maxterm Notation

f = (A+B+C)(A+B+C')(A+B'+C)

f (A,B,C) = $M_0M_1M_2$ or

f (A,B,C) = Π M (0,1,2)

✓ Maxterms present in f correspond with the 0's of f in the truth table.

M and m Relationship

- If the minterm expansion for f (A,B,C) = $m_3 + m_4 + m_5 + m_6 + m_7$, what is the maxterm expansion for f(A,B,C)?
  - ✓Choose those not present in the minterms.
  - – So the Maxterm expansion for f(A,B,C) = $M_0 M_1 M_2$.

Complement of minterm

- Complement of a minterm is the corresponding maxterm.
- Example

  if f = f (A,B,C) = $m_3 + m_4 + m_5 + m_6 + m_7$

✓ $f' = (m_3 + m_4 + m_5 + m_6 + m_7)'$

$= m'_3 \; m'_4 \; m'_5 \; m'_6 \; m'_7$

$= M_3 \, M_4 \, M_5 M_6 M_7$

Find the Minterm Expansion

- ✓ $f(a,b,c,d) = a'(b'+d) + acd'$.

$= a'b' + a'd + acd'$

$= a'b'(c+c')(d+d') + a'd(b+b')(c+c') + acd'(b+b')$

$= a'b'c'd' + a'b'c'd + a'b'cd' + a'b'cd + a'bc'd + a'bcd + abcd' + ab'cd'$

$= \Sigma m(0,1,2,3,5,7,10,14)$

What is the maxterm expansion for f?

Find the Maxterm Expansion

- $f(a,b,c,d) = a'(b'+d) + acd'$.

  $= (a'+cd')(a+b'+d)$; Use $(x+y)(x'+z)=xz+x'y$.

  $= (a'+c)(a'+d')(a+b'+d)$; Use $(x+y)(x+z) = x+yz$.

  $=(a'+bb'+c+dd')(a'+bb'+cc'+d')(a+b'+cc'+d)$

  $=(a'+bb'+c+d)(a'+bb'+c+d')(a'+bb'+c+d')(a'+bb'+c'+d')(a+b'+cc'+d)$

  $=(a'+b+c+d)(a'+b'+c+d)(a'+b+c+d')(a'+b'+c+d')(a'+b+c'+d')(a'+b'+c'+d')(a+b'+c+d)(a+b'+c'+d)$

  $=\Pi M(4,6,8,9,11,12,13,15)$;

  primed = 1, unprimed = 0.

  Note that maxterm = 0.

General Expressions

- n variables (i = 0 to 2^n-1values)

Minterm : $F = \Sigma a_i m_i$

If $a_i = 1$, then minterm m_i exists.

Maxterm : $F = \Pi(a_i + M_i)$;

If $a_i = 1$, then the maxterm does not exist.

✓ Note that $\Sigma a_i m_i = \Pi(a_i + M_i)$

✓ $F' = [\Pi(a_i + M_i)]' = \Sigma a_i' M_i' = \Sigma a_i' m_i = \Pi(a_i' + M_i)$

Incompletely Specified Functions

- Don't care terms.
  - A'B'C and ABC' are "don't care" term. We don't care the value of these terms,whether it is 1 or 0.
  - ✓ Example

    F= A'B'C'+A'BC +ABC = A'B'C' + BC (assign 0 to both X's)

    F = A'B'C'+A'B'C+A'BC+ABC = A'B'+BC (assign 1 to first X and 0 to the second)

    F = A'B'+BC+AB (assign 1 to both X's).

A	B	C	F
0	0	0	1
0	0	1	X
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	X
1	1	1	1

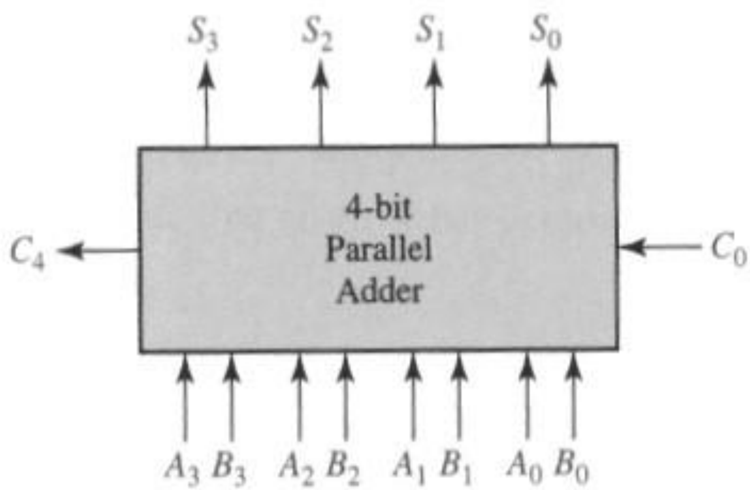
Minterm Expansion for Don't Care

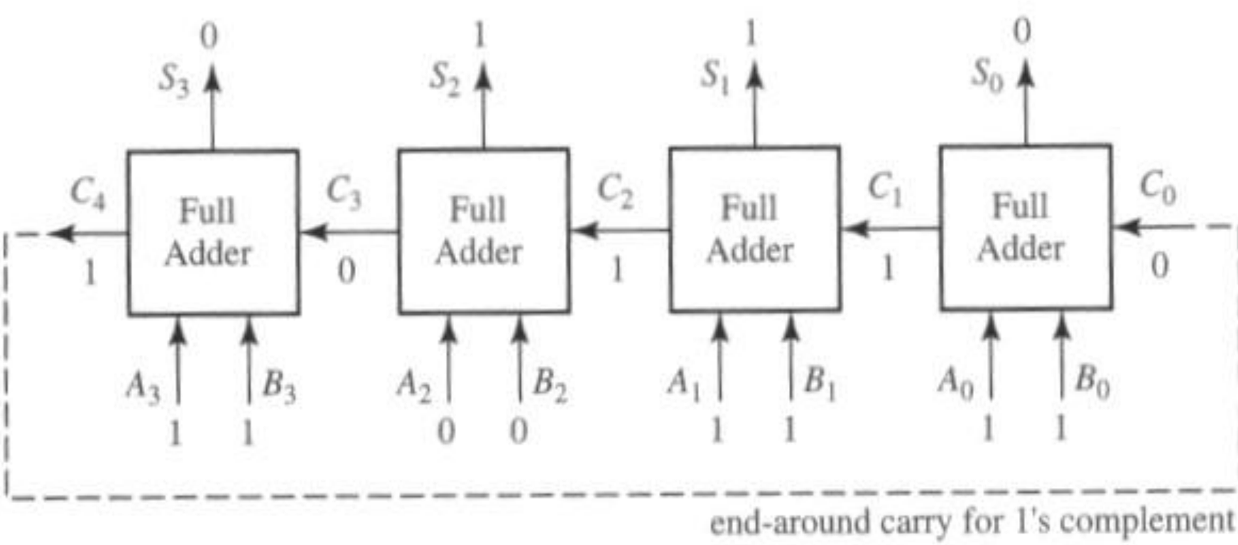
- Example
  - Minterm
    - $F = \Sigma m(0,3,7) + \Sigma d(1,6)$
  - Maxterm
    - $F = \Pi M(2,4,5) \cdot \Pi D(1,6)$

A	B	C	F
0	0	0	1
0	0	1	X
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	X
1	1	1	1

Examples

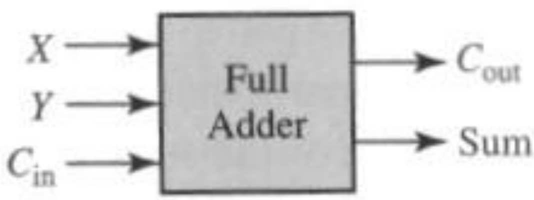
- 1’s complement adder





Full Adder

- One bit



X	Y	C_{in}	C_{out}	Sum
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

$$
\begin{aligned}
Sum &= X'Y'C_{in} + X'YC'_{in} + XY'C'_{in} + XYC_{in} \\
&= X'(Y'C_{in} + YC'_{in}) + X(Y'C'_{in} + YC_{in}) \\
&= X'(Y \oplus C_{in}) + X(Y \oplus C_{in})' = X \oplus Y \oplus C_{in}
\end{aligned}
$$

$$
\begin{aligned}
C_{out} &= X'YC_{in} + XY'C_{in} + XYC'_{in} + XYC_{in} \\
&= (X'YC_{in} + XYC_{in}) + (XY'C_{in} + XYC_{in}) + (XYC'_{in} + XYC_{in}) \\
&= YC_{in} + XC_{in} + XY
\end{aligned}
$$

2’s Complement Adder

- Form 2’s complement for minus operand for subtraction