Lecture 4
Minterm and Maxterm

Conversion of English sentences to Boolean equations.

• Example
  – The alarm will ring iff the alarm switch is on and the door is not closed or it is after 6 PM and the window is not closed.
  – Boolean equation
    • \( Z = AB' + CD' \)
    • If \( Z = 1 \), the alarm will ring.
    • Draw the network. \( Z \) will drive the alarm.
Combination Network Using a Truth Table

• Problem statement
  ❖ Input 3 bits A, B, C = Binary number N. Output f. Output f = 1 if N >= 011 and f = 0 if N < 011.

<table>
<thead>
<tr>
<th>A B C</th>
<th>f</th>
</tr>
</thead>
<tbody>
<tr>
<td>000</td>
<td>0</td>
</tr>
<tr>
<td>001</td>
<td>0</td>
</tr>
<tr>
<td>010</td>
<td>0</td>
</tr>
<tr>
<td>011</td>
<td>1</td>
</tr>
<tr>
<td>100</td>
<td>1</td>
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<tr>
<td>101</td>
<td>1</td>
</tr>
<tr>
<td>110</td>
<td>1</td>
</tr>
<tr>
<td>111</td>
<td>1</td>
</tr>
</tbody>
</table>
Derive Algebraic Expression from Truth Table

• Using $f = 1$ gives the SOP form.

$$f = A'B'C + AB'C' + AB'C + ABC' + ABC = A'B'C + AB' + AB = A'B'C + A = A + BC.$$ 

• Using $f = 0$ gives the POS form.

Maxterms are multiplied together so that if any one of them is 0, $f$ will be 0. See what happens if uses OR.

$$f = (A+B+C)(A+B+C')(A+B'+C) = (A+B)(A+B'+C) = A+BC.$$
Minterm and Maxterm

Minterm

- A minterm of n variables = product of n literals in which each variable appears exactly once either in T or F form, but not in both. (Also known as a standard product term)
- Each minterm has value 1 for exactly one combination of values of variables. E.g. ABC (111) => m₇
- A function can be written as a sum of minterms, which is referred to as a minterm expansion or a standard sum of products.
Minterm/Maxterm

- Three variables

<table>
<thead>
<tr>
<th>Row No.</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>Minterms</th>
<th>Maxterms</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$A'B'C' = m_0$</td>
<td>$A + B + C = M_0$</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>$A'B'C = m_1$</td>
<td>$A + B + C' = M_1$</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>$A'BC' = m_2$</td>
<td>$A + B' + C = M_2$</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>$A'BC = m_3$</td>
<td>$A + B' + C' = M_3$</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>$AB'C' = m_4$</td>
<td>$A' + B + C = M_4$</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>$AB'C = m_5$</td>
<td>$A' + B + C' = M_5$</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>$ABC' = m_6$</td>
<td>$A' + B' + C = M_6$</td>
</tr>
<tr>
<td>7</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>$ABC = m_7$</td>
<td>$A' + B' + C' = M_7$</td>
</tr>
</tbody>
</table>
Minterm Notation

- $f = A'B'C + AB'C' + AB'C + ABC'$ + ABC;

The other way to represent $f$ is:

$f (A,B,C) = m_3 + m_4 + m_5 + m_6 + m_7$

or

$f (A,B,C) = \sum m(3,4,5,6,7)$

Another view,

$f (A,B,C) = 0.m_0 + 0.m_1 + 0.m_2 + 1. m_3$

$+ 1. m_4 + 1. m_5 + 1. m_6 + 1. m_7$

✓ Minterms present in $f$ correspond with the 1’s of $f$ in the truth table.
Maxterm

• Maxterm
  – A maxterm of n variables = sum of n literals in which each variable appears exactly once in T or F from, but not in both.
  – Each maxterm has a value of 0 for exactly one combination of values of variables. E. g. $A + B + C'$ (001) $\Rightarrow M_1$ (the value is 0). Therefore $M_i = m'_i$.
  – A function can be written as a product of maxterms, which is referred to as a maxterm expansion or a standard product of sums.
Maxterm Notation

\[ f = (A+B+C)(A+B+C')(A+B'+C) \]
\[ f (A,B,C) = M_0M_1M_2 \text{ or} \]
\[ f (A,B,C) = \Pi M (0,1,2) \]

✓ Maxterms present in \( f \) correspond with the 0’s of \( f \) in the truth table.
M and m Relationship

• If the minterm expansion for $f(A,B,C) = m_3 + m_4 + m_5 + m_6 + m_7$, what is the maxterm expansion for $f(A,B,C)$?

  ✓ Choose those not present in the minterms.
  – So the Maxterm expansion for $f(A,B,C) = M_0M_1M_2$. 
Complement of minterm

• Complement of a minterm is the corresponding maxterm.

• Example

  if \( f = f(A,B,C) = m_3 + m_4 + m_5 + m_6 + m_7 \)

\[ f' = (m_3 + m_4 + m_5 + m_6 + m_7)' = m'_3 \ m'_4 \ m'_5 \ m'_6 \ m'_7 = M_3 \ M_4 \ M_5 \ M_6 \ M_7 \]
Find the Minterm Expansion

✓ \( f(a,b,c,d) = a'(b'+d) + acd' \).

= \( a'b' + a'd + acd' \)

= \( a'b'(c+c')(d+d') + a'd(b+b')(c+c') + acd'(b+b') \)

= \( a'b'c'd' + a'b'c'd + a'b'cd' + a'b'cd + a'bc'd + a'bcd + abcd' + ab'cd' \)

= \( \Sigma m(0,1,2,3,5,7,10,14) \)

What is the maxterm expansion for \( f \)?
Find the Maxterm Expansion

\[ f(a, b, c, d) = a'(b'+d) + acd'. \]

\[ = (a'+cd')(a+b'+d); \quad \text{Use} \]
\[ (x+y)(x'+z)=xz+x'y. \]
\[ = (a'+c)(a'+d')(a+b'+d); \quad \text{Use} \]
\[ (x+y)(x+z) = x+yz. \]
\[ =(a'+bb'+c+dd')(a'+bb'+cc'+d')(a+b' +cc'+d) \]
\[ =(a'+bb'+c+d)(a'+bb'+c+d')(a'+bb'+c'+d')(a+b'+cc'+d) \]
\[ =(a'+b+c+d)(a'+b'+c+d)(a'+b+c+d')(a+b'+c'+d')(a+b'+c+d) \]
\[ =\Pi M(4,6,8,9,11,12,13,15); \]

primed = 1, unprimed = 0.

Note that maxterm = 0.
General Expressions

• n variables (i = 0 to $2^n-1$ values)

Minterm: $F = \Sigma a_i m_i$
If $a_i = 1$, then minterm $m_i$ exists.

Maxterm: $F = \Pi (a_i + M_i)$;
If $a_i = 1$, then the maxterm does not exist.

✓ Note that $\Sigma a_i m_i = \Pi (a_i + M_i)$
✓ $F' = [\Pi (a_i + M_i)]' = \Sigma a_i'M_i' = \Sigma a_i'm_i = \Pi (a_i' + M_i)$
Incompletely Specified Functions

• Don’t care terms.
  – A’B’C and ABC’ are “don’t care” term. We don’t care the value of these terms, whether it is 1 or 0.

✓ Example

\[
F = A’B’C’+A’BC +ABC = A’B’C’ + BC
\]
(assign 0 to both X’s)

\[
F = A’B’C’+A’B’C+A’BC+ABC = A’B’+BC
\]
(assign 1 to first X and 0 to the second)

\[
F = A’B’+BC+AB
\]
(assign 1 to both X’s).

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>F</th>
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<tbody>
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Minterm Expansion for Don’t Care

- Example
  - Minterm
    - \( F = \sum m(0,3,7) + \sum d(1,6) \)
  - Maxterm
    - \( F = \Pi M(2,4,5) \cdot \Pi D(1,6) \)

<table>
<thead>
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<th>A</th>
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<tbody>
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Examples

- 1’s complement adder
Full Adder

- One bit

\[
\text{Sum} = X'Y'C_{in} + X'YC_{in}' + XY'C_{in}' + XYC_{in}
\]

\[
= X'(Y'C_{in} + YC_{in}') + X(Y'C_{in}' + YC_{in})
\]

\[
= X'(Y \oplus C_{in}) + X(Y \oplus C_{in})' = X \oplus Y \oplus C_{in}
\]

\[
C_{out} = X'YC_{in} + XY'C_{in} + XYC_{in}' + XYC_{in}
\]

\[
= (X'YC_{in} + XYC_{in}) + (XY'C_{in} + XYC_{in}) + (XYC_{in}' + XYC_{in})
\]

\[
= YC_{in} + XC_{in} + XY
\]
2’s Complement Adder

- Form 2’s complement for minus operand for subtraction