Lecture 3
Algebraic Simplification

- Consensus Theorem
  - XY + X'Z + YZ =
  - XY + X'Z
  - YZ, which is referred to as consensus term, is redundant.
- Example
  - The consensus term for *abd* and *b'de'* is *ade'*.
- The dual form of the consensus theorem.
  - (X+Y)(X'+Z)(Y+Z) = (X+Y)(X'+Z)

Consensus Term

- Example
  - A’C’D + A’BD + BCD +ABC +ACD’
    - BCD can be eliminated by (A’BD, ABC) pair. But we can do it the other way.
  - A’C’D + A’BD + BCD +ABC +ACD’
    - (A’C’D, BCD) => A’BD
    - (BCD, ACD’) => ABC

Add a Consensus Term

- Example: Add consensus term to eliminate other terms.
  - F = ABCD + B'CDE + A'B' + BCE'
    - ABCD + B'CDE => ACDE, add this to F
  - F = ABCD + B'CDE + A'B' + BCE' + ACDE
    - (BCE' , ACDE) => ABCD
    - (A'B' , ACDE) => B'CDE

Simplification Using Algebraic Methods

- Combining terms.
  - $XY + XY' = X$
- Example:
  - $abc'd' + abcd' = abd'$
  - $ab'c + abc + a'bc = ab'c + abc + abc + a'bc = ac + bc.$
  - $(a+bc)(d+e') + a'(b'+c')(d+e') = d + e'.$

Simplification Using Algebraic Methods

- Eliminating terms.
  - X + XY = X
  - XY + X'Z + YZ = XY + X'Z
- Example:
  - a'b + a'bc = a'b
  - a'bc' + bcd + a'bd = a'bc'+ bcd

Simplification Using Algebraic Methods

- Eliminating literals
  - $X + X'Y = X + Y$
- Example:
  - $A'B + A'B'C'D' + ABCD'$

    $= A'(B + B'C'D') + ABCD'$

    $= A'(B + C'D') + ABCD'$

    $= B(A' + ACD') + A'C'D$

    $= B(A' + CD') + A'C'D$

    $= A'B + BCD' + A'C'D'$

Simplification Using Algebraic Methods

- Adding Redundant Terms
  - Add XX'
  - Multiply (X+X')
  - Add YZ to XY + X'Z
- Example:
  - *WX* + XY + *X'Z*' + WY'Z'

    = WX + XY + X'Z' + WY'Z' + WZ'

    (add WZ')

    = WX + XY + X'Z' + WZ'

    (Using X + XY = X)

    = WX + XY + X'Z'

Simplification Example Using Dual Theorems

- Example
  - (A’ + B’ + C’)(A’ + B’ + C) (B’ + C)(A +C) *(A + B + C)* ---*Eq.(1)*
  - (A’ + B’ + C’)(A’ + B’ + C) = (A’ + B’), Why? (Dual XY’ + XY = X)
  - *(A + B + C)* can be eliminated. Why?

    (A+C) (A+B+C) => dual X + XY = X, so (A+C) (A+B+C) = A+C.
  - Therefore,

    Eq. (1) = (A’ + B’)(B’+C)(A+C) = (A’+ B’)(A +C)? Why?

    (B’ +C) is the consensus term.

Proving Validity

- Proving if an equation is valid for all combinations of values of variables
  - Methodology
    - Using a truth table => Tedious.
    - Heuristic
      - Manipulate one side of equation to see if it matches the other side.
      - Reduce both side to the same expression.
      - Perform reversible operation on both sides.
        - » Complement on both sides
        - » Multiplication and addition of the same term to both sides are not permissible. (Why? Division and subtraction are not defined for Boolean algebra.)

Validation Example

- Show that the following equation is valid.
  - A'BC'D + (A'+BC)(A+C'D') +BC'D + A'BC'

    = ABCD + A'C"D' + ABD + ABCD" + BC'D.

  – For the left side, we reduce it:

    LS = (A' + BC)(A+C'D') + BC'D + A'BC'; *A'BC'D + BC'D = BC'D*

    LS = A'C'D' + ABC + BC'D + A'BC'; *(X+Y)(X'+Z) = XZ +X'Y*

    LS = A'C'D' + ABC + BC'D ; *(A'C'D', BC'D)*

    RS = ABC + A'C'D' + ABD + BC'D; *(ABCD + ABCD' = ABC)*

    RS = ABC + A'C'D' + BC'D; *(ABC, BC'D)*

  – **So the original equation is valid.**

Cancellation Law is not True

- Cancellation law is not true for Boolean algebra, i.e., If x +y = x + z, then y=z
  - (let x = 1, y = 0, z= 1)
- Cancellation law for multiplication is not valid for Boolean algebra, i. e. if xy = xz, then y = z.
  - **Let x = 0, y = 0, z = 1.**
  - This is why in proving the validity of an equation, although adding the same term to both sides of the equation leads to a valid equation, but the reverse operation (canceling or subtracting a term from both sides) does not lead to a valid equation. Therefore, when we try to prove that an equation is valid, it is not permissible to add or to multiply both sides by the same expression, because the addition or the multiplication operations are not reversible.