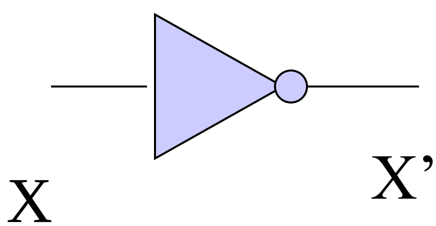
Lecture 2
Boolean Algebra Basic

- Algebra
  - An algebra A is a mathematics theory involving a set of elements S and a set of operations O that act on the members of S.
  - The smallest Boolean algebra contains two values, 0 and 1.
  - If X is a Boolean variable, then either X = 0 or X = 1.
  - 1 can mean high voltage or True while 0 might correspond to low voltage or False.

Basic Operations

- NOT (Complement, inversion)

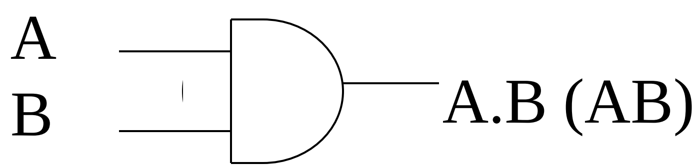


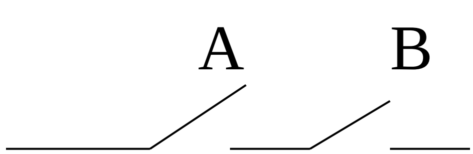
X	X'
0	1
1	0

Truth table

Basic Operations-cont.

- AND (Boolean multiplication)



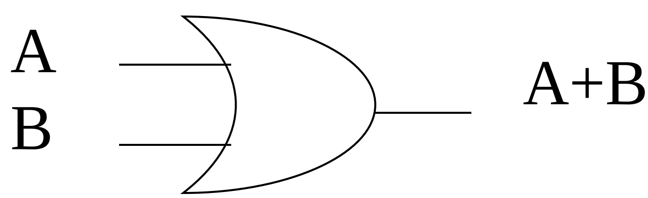


Open: 0
Close: 1

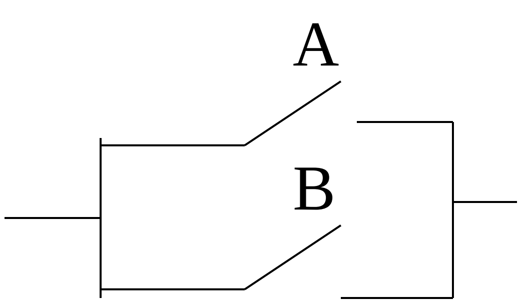
A B	A.B
0 0	0
0 1	0
1 0	0
1 1	1

Basic Operations-cont.

- OR (Boolean addition, inclusive OR)



A B	A + B
0 0	0
0 1	1
1 0	1
1 1	1



Open: 0
Close: 1

Boolean Expressions

- Expressions
- Examples:

  AB' + C (1)

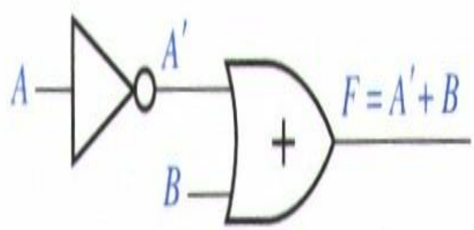
  [A(C+ D)]' + BC (2)

  – Each expression corresponds to a network of logic gates.
    - Exercise: Draw the logic gate network for (1) and (2).

  – In (2), there are 4 variables and 5 literals.
- Operation order:
  – Parenthesis, NOT-> AND-> OR

Literals and Truth Table

- Literals
  - When an expression is realized using logic gates, each literal in the expression corresponds to a gate input.



(a)

A B	A′	F = A′ + B
0 0	1	1
0 1	1	1
1 0	0	0
1 1	0	1

(b)

2-input Network and Truth Table

Literals and Truth Table

- Truth table
  - Specify the values of the variables in the expression.
  - n variables have 2^n different combinations of values of the variables.

A	B	C	B'	AB'	$AB' + C$	$A + C$	$B' + C$	$(A + C)(B' + C)$
0	0	0	1	0	0	0	1	0
0	0	1	1	0	1	1	1	1
0	1	0	0	0	0	0	0	0
0	1	1	0	0	1	1	1	1
1	0	0	1	1	1	1	1	1
1	0	1	1	1	1	1	1	1
1	1	0	0	0	0	1	0	0
1	1	1	0	0	1	1	1	1

Basic Theorems

- Operation with 0 and 1

  $X + 0 = X$

  $X + 1 = 1$

  $X \cdot 1 = X$

  $X \cdot 0 = 0$

- Idempotent laws

  $X + X = X$

  $X \cdot X = X$

- Involution law

  $(X')' = X$

- Laws of complementarity

  $X + X' = 1$

  $X \cdot X' = 0$

Basic Theorems-cont.

- Commutative laws

$$X + Y = Y + X$$

$$XY = YX$$

- Associative laws

$$(X+Y) + Z = X + (Y + Z) = X + Y + Z$$

$$(XY)Z = X(YZ) = XYZ$$

- Distributive laws

$$X(Y + Z) = XY + XZ$$

$$X + YZ = (X+Y)(X+Z)$$

Basic Theorems-cont.

- Simplification theorems

XY + XY’ = X

✓ (X+Y)(X+Y’) = X

X + XY = X

✓ X(X+Y) = X

(X+Y’)Y = XY

✓ XY’ + Y = X + Y

Example: Simplification

Z = (AB+C)(B’D+C’E’) + (AB+C)’= (AB+C)’ + B’D+C’E’

Basic Theorems-cont.

- Theorem for multiplying out and factoring
  - Use distributive law.

✓ $(X+Y)(X+Z) = X + YZ$

Example: $(A+BC)(A+D+E)$

$= A + BC(D+E) = A + BCD + BCE$.

Proof of Theorem or Law

- Using truth table
  - Associative law
    - (A+B) + C = A + B + C
    - What is the difference?
    - # of fan-in
    - Potential speedup
- Using algebra manipulation
    - A +A = A
- Using Principal of Duality
  - ✓ Principal of duality: Any theorem or identity in switching algebra remains true if 0 and 1 are swapped and, $\cdot$ and + are swapped.
  - ✓ Example: If $X + XY = X \cdot 1$, then

    $X \cdot (X+Y) = X + 0 = X$

Some Properties

- In Boolean algebra, AND (OR) operation distributes over OR(AND).
  - ✓ X(Y+Z) = XY + XZ
  - ✓ A+BC = (A+B)(A+C)
- Application of simplification

  Example: (A + B'C + D + EF)(A+B'C+ (D+EF)');

  - ❑ View A+B'C = X, D + EF = Y; so this is (X+Y)(X+Y') = X

SOP

- An expression is said to be in **sum of products** form when all products are products of single variables only. Use frequently.
- Example

A’B + CD’E + AC’E’	Yes.
(A+B)CD + EF	No.
ACD +BCD +EF	Yes.

- Use Multiplying out to get SOP

  (A+BC)(A+D+E) =

  A + BC(D+E) = A + BCD +DCE.

  - Using (X+Y)(X+Z) = X + YZ

POS

- An expression is said to be in **product of sums** form when all sums are sums of a single variable. Not use as often as SOP form in industry.
- Example

(A+B')(C+D'+E)	Yes.
(A+B')(C+D'+E)F	Yes.
(A+B)(C+D) + EF	No.

- Use Factoring to get POS

A + B'CD = (A + B') (A+ CD); first factoring.

= (A+B')(A+C)(A+D); second factoring.